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*The Brown measure of the sum of a free random variable and Voiculescu's circular element or its elliptic deformation*

The circular element is the most important example of non-normal random variable used in free probability, and its Brown measure is the uniform measure in the unit disk. The circular element has connection to asymptotics of non-normal random matrices with i.i.d. entries. We obtain a formula for the Brown measure of the addition  $x_0 + c$  of an arbitrary free random variable  $x_0$  and circular element  $c$ , which is known to be the limit empirical spectral distribution of deformed i.i.d. random matrices.

Generalizing the case of circular and semi-circular elements, we also consider  $g$ , a family of elliptic deformations of  $c$ , that is  $*$ -free from  $x_0$ . Possible degeneracy then prevents a direct calculation of the Brown measure of  $x_0 + g$ . We instead show that the whole family of Brown measures of operators  $x_0 + g$  are the push-forward measures of the Brown measure of  $x_0 + c$  under a family of self-maps of the complex plane, which could possibly be singular. We calculate density formulas for various interesting examples.

This work generalizes previous results of Bordenave-Caputo-Chafaï, Hall-Ho, and a joint work with Ho. The main results offer potential applications to various deformed random matrix models. Our method is based on a Hermitian reduction and subordination functions in free probability.