Applied geometric analysis
Analyse géométrique appliquée
(Org: **Chunhua Ou** and/et **Jie Xiao** (Memorial University))

GOONG CHEN, Mathematics Department, Texas A&M University
Biological shapes, modal analysis, and visualization of motion

In this talk, we will discuss some of the shapes of biological creatures such as dinosaurs, camels, horses, etc., and compute and visualize their eigenfunctions of elastodynamics, which are often called normal modes of vibration of those elastic solids. We compute and visualize such modes by using LS-DYNA as our software platform, and try to interpret their significance in terms of the reality of the lives of these animals. Furthermore, we try to incorporate those modes together with the (artistic rendering) software package Blender to visualize the motions of those animals. We hope that our work can help understand the interplay between shapes, mechanics/dynamics, and biological behaviors.

CRISTIAN ENACHE, American University of Sharjah
On some monotonicity properties of the p-torsional rigidity

For a bounded domain $\Omega \subset \mathbb{R}^N$, $N \geq 2$ and a real number $p > 1$, we denote by u_p the p *-torsion function* on Ω, that is the solution of the *torsional creep problem* $\Delta_p u = -1$ in Ω, $u = 0$ on $\partial\Omega$, where $\Delta_p := div(|\nabla u|^{p-2}\nabla u)$ is the p-Laplace operator. In this talk we are going to present some monotonicity properties for the p *-torsional rigidity* on Ω, defined as $T_p(\Omega) := \int_\Omega u_p dx$, and for $p \to T(p;\Omega) := |\Omega|^{p-1} T_p(\Omega)^{1-p}$.

SURESH ESWARATHASAN, Dalhousie University
Eigenvalues of ellipsoids close to spheres

We study the spectrum of the Laplace-Beltrami operator on $2D$ ellipsoids . For ellipsoids that are "close" to the $2D$ sphere, we use analytic perturbation theory (à la F. Rellich and M. Berger) to estimate the eigenvalues up to two orders with respect to the "closeness" parameter. We show that for biaxial ellipsoids sufficiently "close" to the sphere, the first N^2 eigenvalues have multiplicity at most two and characterize those that are simple. For a class of triaxial ellipsoids which are not biaxial, we prove the first sixteen eigenvalues are simple.

This is joint work with Theodore Kolokolnikov (Dalhousie University).

YUNHUI HE, The University of British Columbia
Local Fourier analysis and its application to multigrid for elliptic optimal control problems

In this talk, we first give a brief introduction to local Fourier analysis (LFA). Then we describe applications of LFA to multigrid for control problems, whose discrete linear systems have a saddle-point structure. We propose a novel Braess-Sarazin multigrid relaxation scheme for finite element discretizations of distributed control problems, where we use the stiffness matrix obtained from the five-point finite difference method for the Laplacian as a smoother for the linear system with a mass matrix coefficient arising in the saddle-point system. To solve elliptic sparse optimal control problems with control constraints, discretized by a finite difference method, we study and compare two multigrid relaxation schemes with coarsening by a factor of two, three, and four. We derive LFA optimal smoothing factor for a well-known collective Jacobi relaxation (CJR) scheme. This analysis reveals that the optimal relaxation parameters depend on the mesh size and regularization parameters. To improve CJR, we propose and analyze a new mass-based Braess-Sarazin relaxation scheme for the finite difference discretization, and prove to provide smaller smoothing factors than the CJR scheme for some cases. These schemes are successfully extended to control-constrained cases through the semi-smooth Newton method. Numerical examples are presented to validate our theoretical observations.

ALINA STANCU, Concordia University
The fundamental gap of convex domains in hyperbolic space revisited

The difference between the first two eigenvalues of the Dirichlet Laplacian on convex domains of $\mathbb{R}^n$ and, respectively $\mathbb{S}^n$, satisfies the same strictly positive lower bound depending on the square of the diameter of the domain. In work with collaborators, we have found that the gap of the hyperbolic space on convex domains behaves strikingly different even if a stronger notion of convexity is employed. This is very interesting as many other features of first two eigenvalues behave in the same way on all three spaces of constant sectional curvature. We will discuss the possibility of a different lower bound on the fundamental gap in the hyperbolic space. (Based on joint work with T.Bourni, J.Clutterbuck, H.Nguyen, G.Wei and V.Wheeler.)

JUN-CHENG WEI, University of British Columbia
Jacobi-Toda systems and interfaces with higher multiplicities

We consider solutions to Allen-Cahn equation

$$\epsilon^2 \Delta u + u - u^3 = 0$$

with bounded energy

$$E_\epsilon(u) = \frac{\epsilon}{2}\int |\nabla u|^2 + \frac{1}{4\epsilon}\int (1-u^2)^2 \leq C$$

It is known that the zero level sets converges $\sum_j m_j \Gamma_j$, where each Γ_j is a minimal surface. When $m_j = 1$, the minimal surface is simple and one can show $C^{1,\alpha}$ or $C^{2,\alpha}$ convergence. In this talk we will study the most difficult case, i.e., $m_j \geq 2$, higher multiplicities. Chosh-Matoulidis (Annals of Math 2019) showed that when $m_j = 2, 3 \leq d \leq 7$, there must exist a nontrivial positive Jacobi field. We first derive a second order condition for interfaces with higher multiplicities, and use it to construct solutions with multiplicity 2. Then we give a complete classification of geodesic nets with higher multiplicity in the two-dimensional case. We construct and compute the precise Morse index in terms of m_j and the number of intersections. The key role played in the above analysis is the behavior of solutions to Jacob-Toda systems. (Joint work with F. Pacard and Y. Liu.)

ZHICHUN ZHAI, MacEwan University
Fractional Extension/Trace Inequalities via Caffarelli-Silvestre Extension

Let $u(x,t)$ be the Caffarelli-Silvestre extension of a smooth function $f(x)$ from $\mathbb{R}^n$ to $\mathbb{R}^{n+1}_+ := \mathbb{R}^n \times (0,\infty)$. We characterize nonnegative measures μ on $\mathbb{R}^{n+1}_+$ such that $f(x) \longrightarrow u(x,t)$ induces bounded embeddings from $L^p(\mathbb{R}^n)$ to $L^q(\mathbb{R}^{n+1}_+;\mu)$ or from $\dot{\Lambda}^{p,q}_\beta(\mathbb{R}^n)$ to $L^{p_0,q_0}(\mathbb{R}^{n+1}_+;\mu)$ via a newly introduced L^p-capacity associated with the Caffarelli-Silvestre extension or the fractional Besov capacities. Then, we establish the fractional anisotropic trace version of the Sobolev inequalities, logarithmic Sobolev inequalities and Hardy inequalities.

This talk is based on joint work with Rui Hu, Pengtao Li and Shaoguang Shi.