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**Symplectic geometry and moduli spaces**  
**Géométrie symplectique et espaces de modules**

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**TOM BAIRD**, Memorial University

*Harmonic forms on the Chen-Teo gravitational instanton*

The famous no-hair theorem in general relativity asserts that Kerr blackholes are the only non-trivial, stationary, axisymmetric, asymptotically flat solutions to the vacuum Einstein equations. A Riemannian/Euclidean version the no-hair theorem was conjectured in the 70s. Namely, that the Euclidean Kerr metrics are the only non-trivial asymptotically flat gravitational instantons admitting a 2-torus symmetry.

A surprising counterexample to this conjecture was discovered in 2011 by Chen and Teo. First constructed using the inverse scatter method, the metric has recently been proven to be conformal to a Kaehler metric on a toric variety (thus fitting with the theme of this session). In this talk I will survey some of these recent developments and discuss my work with Kunduri classifying L2-harmonic forms on the Chen-Teo gravitational instanton.

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**PETER CROOKS**, Northeastern University

*Symplectic reduction and sheets*

Let  $\mathfrak{g}$  be a finite-dimensional semisimple Lie algebra over  $\mathbb{C}$ . Given a non-negative integer  $k$ , one may consider the locus  $\mathfrak{g}_k \subset \mathfrak{g}$  of elements with centralizer dimension equal to  $k$ . A locally closed subvariety of  $\mathfrak{g}$  is called a *sheet* if it is an irreducible component of  $\mathfrak{g}_k$  for some non-negative integer  $k$ . These subvarieties are ubiquitous in Lie-theoretic Poisson geometry.

I will give an overview of sheets and their incarnations in Poisson geometry and representation theory. This will lead to a description of ongoing work with Maxence Mayrand, in which we consider symplectic reduction along a sheet.

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**SHENGDA HU**, Wilfrid Laurier University

*Degree of a generalized holomorphic bundle*

We discuss a notion of degree for a generalized holomorphic vector bundle over a generalized Kahler manifold, using generalized connections that are analogues of Chern connections in the classical situation. The resulting degree fits well with the analogue of Hermitian-Yang-Mills equation, as proposed by Hitchin in 2011, in this context. Some general consequences of our constructions will be discussed as well.

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**YUCONG JIANG**, University of Toronto

*Dirac structures in generalized Kähler geometry*

Abstract: Generalized Kähler (GK) structures were discovered by Gates, Hull, and Roček in 1984 during their study of N=(2,2) supersymmetric nonlinear sigma models and later on put into the framework of generalized complex geometry by Gualtieri in his 2003 thesis. In this talk, We will first review the construction of holomorphic Dirac structures in GK geometry and explain how they can be used to study the real and holomorphic Poisson structures occurring in a GK manifold. We will use tools from the theory of Lie groupoids, and see how symplectic groupoids, presymplectic groupoids and Poisson groupoids enter naturally in GK geometry and what they will imply for the underlying GK structure. Based on joint work Marco Gualtieri.

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**RUXANDRA MORARU**, University of Waterloo

*Co-Higgs bundles on Poisson surfaces*

Co-Higgs bundles on a complex manifold  $M$  are given by pairs  $(E, \phi)$  consisting of a holomorphic vector bundle  $E$  on  $M$  together with a Higgs field  $\phi \in H^0(M, \text{End}(E) \otimes TM)$  that satisfies certain integrability conditions. In particular, co-Higgs bundles correspond to generalized holomorphic bundles on complex manifolds. They also give rise to a special class of holomorphic Poisson structures on the projective bundles  $\mathbb{P}(E)$ . Co-Higgs bundles were first studied by S. Rayan on Riemann surfaces and  $\mathbb{C}\mathbb{P}^2$  in his thesis, where he also gave a non-existence theorem for stable, traceless rank-2 co-Higgs bundles on K3 and general-type surfaces. In this talk, we consider co-Higgs bundles on all compact holomorphic Poisson surfaces. We give necessary and sufficient conditions for the existence of stable, traceless rank-2 co-Higgs bundles on all compact holomorphic Poisson surfaces as well as a complete classification of such bundles on some surfaces.

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**DOUG PARK**, University of Waterloo

*Geography problem for simply connected symplectic 4-manifolds*

We will discuss the following *geography* problem: Given a pair of integers  $(a, b)$ , is there a simply connected minimal symplectic 4-dimensional manifold  $M$  such that the Euler characteristic of  $M$  is  $a$ , and the signature of  $M$  is  $b$ ? Here, we say that a symplectic 4-dimensional manifold is *minimal* if it is not diffeomorphic to a symplectic blow-up of some other symplectic 4-dimensional manifold. If we restrict our attention to *nonspin* 4-dimensional manifolds, then this problem has been completely solved in the case when the signature variable  $b$  is negative. We will discuss the remaining case when  $b$  is nonnegative, highlighting the speaker's recent joint works with his collaborators.

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**SHIYU SHEN**, University of Toronto

*Complex K-theory of the moduli space of Higgs bundles*

The moduli spaces of semistable  $SL_n$  and  $PGL_n$  Higgs bundles form dual abelian fibrations over (generic locus of) an affine space called the Hitchin base. Motivated by the Hausel-Thaddeus mirror symmetry picture, we construct an isomorphism between the complex K-theory of the two moduli spaces on the spectra level. As part of our proof, we show both K-theory groups are torsion-free. This is joint work in progress with Michael Groechenig.