Relative Homology and Persistence Theory
Homologie relative et théorie de la persistance
(Org: **Véronique Bazier-Matte** (University of Connecticut), **Thomas Brüstle** (Sherbrooke) and/et **Eric Hanson** (UQAM))

BENJAMIN BLANCHETTE, Université de Sherbrooke
Homological invariants in persistence theory

Let $\mathcal{X}$ be a finite set of indecomposable modules over a given poset $\mathcal{P}$. A homological invariant relative to $\mathcal{X}$ can be seen as an approximation by a linear combination of modules in $\mathcal{X}$, which is compatible with an exact structure. We define and study these invariants, using relative homology. In particular, we compare them with known invariants such as the Hibert function, the (generalized) rank invariants, the signed barcode and compressed multiplicities. We show that the set of single-source spread modules yields a homological invariant closely related to the concept of persistence, and is strictly finer than the classical rank invariant.

This is joint work with Thomas Brüstle and Eric J. Hanson.

HARM DERKSEN, Northeastern University
Multiparameter Landscapes

The persistence landscape was introduced by Bubenik as an alternative to the barcode or persistence diagram. Although it encodes the same information, it has particularly nice properties. Vipond then defined a multiparameter generalization of persistence landscape which encodes the same data as the rank invariant. We generalize Vipond's construction to a multiparameter persistence landscape that has the same good properties, but has more flexibility in the data it encodes. Namely, for any fixed path connected shape in $\mathbb{R}^m$, we produce a persistence landscape associated to a given persistence module M that encodes the restriction of M to all regions similar to that shape. This is joint work with Ryan Kinser.

MARTIN FRANKLAND, University of Regina
Multiparameter persistence modules in the large scale

A persistence module with d discrete parameters is a diagram of vector spaces indexed by the poset $\mathbb{N}^d$. If we are only interested in the large scale behavior of such a diagram, then we can consider two diagrams equivalent if they agree outside of a "negligeable" region. In the 2-dimensional case, we classify the indecomposable diagrams up to finitely supported diagrams. In higher dimension, we partially classify the indecomposable diagrams up to suitably finite diagrams.

Along the way, we classify the tensor closed Serre subcategories of the category of finitely generated d-parameter persistence modules: they are in bijection with the simplicial complexes on d vertices. This is joint work with Don Stanley.

IVO HERZOG, The Ohio State University
Eklof's Lemma in Ideal Approximation Theory

We will describe 1) the origin and motivation behind Ideal Approximation Theory; 2) its analogy with classical Approximation Theory in the study of complete ideal cotorsion pairs; 3) its context, the exact category of mono-epi extensions of arrows; and then 4) some recent results and applications related to the ideal version of Eklof's Lemma. The talk will be based on joint work with S. Estrada, X.H. Fu, and S. Odabasi.

MANUEL CORTÉS IZURDIAGA, University of Málaga
Ziegler partial morphisms and approximations in exact categories

In the eighties, Ziegler introduced, using the language of model theory, the notions of partial morphism, partial isomorphism and small extension in the category of modules over a not necessarily commutative ring R, in order to study (and extend) the pure-injective hull of a module. These notions were later characterized by Monari-Martinez in terms of systems of equations.

In the talk we see that an exact category is the natural setting to study these notions, and that they are related with approximations in the category.

The talk is based on joint work with Pedro Guil, Berke Kalebogaz and Ashish Srivastava.

WOOJIN KIM, Duke University

Persistence diagrams via limit-to-colimit maps and Möbius inversions

The persistence diagram (equivalently barcode) has been one of the most prevalent objects in topological data analysis (TDA) as an object which summarizes features of a persistence module. With the goal of adapting the idea behind persistent homology to the study of wider types of data (e.g. time-varying point clouds), variants of the indexing set of persistence modules inevitably occur, leading for example to multiparameter persistence and zigzag persistence. However, it is not always evident how to define a notion of persistence diagram for such variants.

This talk will introduce a generalized notion of persistence diagram for many of such variants which arises through exploiting both the principle of inclusion-exclusion from combinatorics, and the notion of (co)limit from category theory. We describe how this resulting generalized persistence diagram subsumes some other well-known invariants of 2-parameter persistence modules and how it mediates between 2-parameter persistence and zigzag persistence. This talk is based on joint work with Nate Clause, Tamal Dey, Facundo Mémoli, and Samantha Moore.

EZRA MILLER, Duke University

Homological algebra over partially ordered real vector spaces

Using persistent homology to model shapes of embedded planar graphs composed of fruit fly wing veins requires homological algebra for modules over partially ordered real vector spaces. Basic finiteness assumptions for usual commutative algebra and relative homological algebra are too restrictive for persistence in this continuous multiparameter setting. This talk outlines a suitable alternative finiteness condition that robustly encodes topological tameness – which can reasonably be assumed to occur in persistence modules arising from data – in equivalent combinatorial, algebraic, and homological ways, notably including finiteness of homological dimension in a relative homological sense. The motivations and definitions will be explained from scratch, ending with a comparison to ordinary homological dimension over real-exponent polynomial rings.

CHARLES PAQUETTE, Royal Military College of Canada

Representation theory of poset quivers

This is a work in progress with Job. D. Rock and Emine Yildirim. Given a quiver Q (possibly infinite), we replace each arrow by a linearly ordered set to get an object that we call poset quiver. This gives rise to the path category of this poset quiver. We can take the quotient of this category by an (weakly admissible) ideal, and consider the pointwise-finite representations (or modules) over this. Using Crawley-Boevey decomposition theorem for representations of linearly ordered sets (barcode decomposition), we see how the representation theory in this setting is controlled by the interval modules together with representations of some associated quivers to Q. We further study some homological properties in this setting, in particular how to construct the projective modules, and get new hereditary categories.

ANNA SCHENFISCH, Montana State University

The Algebraic K-Theory of Zig-Zag Persistence Modules

In this talk, we will first see how persistence modules (a primary tool in topological data analysis) have a natural home in the setting of stratified spaces and constructible cosheaves. In particular, we focus on zig-zag modules, which correspond to one-parameter filtrations. We then outline how the algebraic K-theory of zig-zag modules can be computed via an additivity result, aided by an equivalence between the category of zig-zag modules and the combinatorial entrance path category on a stratified $\mathbb{R}$. Once equipped with the K-theory of zig-zag modules, we see other one-parameter topological summaries (such as Euler characteristic curves) as classes of K_0.

MARKUS SCHMIDMEIER, Florida Atlantic University
Symmetry and Conservation for Poset Representations

We study representations of doubly infinite posets based on certain vertex sets of the form $\mathbb{Z} \times F$ where F is finite. Symmetries of the poset like translation, reflection and rotation give rise to categorical properties and to distinguished classes of objects. In settings where the representation theory is understood, we study (generalized) rank modules and the rank decomposition as introduced by Botnan, Oppermann and Oudot.