Geometry of eigenfunctions and random fields
Géométrie des fonctions propres et champs aléatoires
(Org: **Graham Cox** (Memorial), **Suresh Eswarathasan** (Dalhousie) and/et **Dmitry Jakobson** (McGill))

SAMUEL AUDET-BEAUMONT, Université Laval
Constructing surfaces with large first Steklov eigenvalue multiplicity

Recently, the study of the behavior of Steklov eigenvalues' multiplicity has seen great advances. These new approaches use both refining of existing methods coming from the study of the Laplacian and interesting novel ideas. Notably, during the last decade, the works of A. Fraser, R. Shoen, P. James, M. Karpukhin, G. Kokarev and I. Polterovich brought multiple results on upper bounds of the multiplicity. These bounds rely on the topology and the number of boundary components of studied surfaces. This raises the following question: is there a universal bound on the multiplicity, independent of these two properties? To disprove it would be to show the existence of surfaces with first nonzero Steklov eigenvalue of arbitrarily large multiplicity.

The objective of this talk is to provide such a construction. The method is based on a technique, developed by Bruno Colbois to get a similar result in the context of the Laplacian eigenvalues. The main idea is to build a surface around the Cayley graph of a given group to get a specific subgroup of isometry. The goal would then be to use the irreducible representations of this subgroup in conjunction with the surface's finer geometry to show that the first non-trivial eigenspace is of large dimension.

THOMAS BECK, Fordham University
Nodal set estimates for perturbed rectangles

We will discuss properties of the nodal sets of low energy Dirichlet eigenfunctions in curvilinear rectangles. In the unperturbed case for a rectangle, the eigenfunctions of interest are the second, where the nodal set is a straight line, and the first eigenfunction with the nodal set containing a crossing, where the nodal set divides the rectangle into four nodal domains. We will describe which properties of the nodal sets and nodal domains are stable under perturbations of the rectangle, and provide quantitative estimates on the slope and curvature of the perturbed nodal sets. In particular, we will show that there is a criterion on how to perturb one side of the rectangle to open the nodal set, resulting in a reduction of the number of nodal domains, and give a sharp estimate on the size of the opening. This is joint work with Yaiza Canzani, Marichi Gupta, and Jeremy Marzuola.

JADE BRISSON, Université Laval
Tubular excision and Steklov eigenvalues

In this talk, I will present a new result concerning the asymptotic behaviour of Steklov eigenvalues for a family of domains obtained by performing small tubular excision around a closed connected submanifold. This submanifold, of positive codimension, is taken in a closed manifold. I will present some applications of this result to isoperimetric type problems and I will give the main ideas behind the proof.

MADELYNE BROWN, University of North Carolina at Chapel Hill
Fourier coefficients of restricted eigenfunctions

We will discuss the growth of Laplace eigenfunctions on a compact, Riemannian manifold when restricted to a submanifold. We analyze the behavior of the restricted eigenfunctions by studying their Fourier coefficients with respect to an arbitrary orthonormal basis for the submanifold. We give an explicit bound on these coefficients depending on how the defect measures for the two collections of functions, the eigenfunctions and the orthonormal basis, relate.

MEHDI EDDAOUDI, Laval University
On the gap between consecutive eigenvalues

The main objective of this talk will focus on the universal inequalities of eigenvalues on a riemannian manifold. This subject emerged following the work initiated by Payne, Polya and Weinberger where they studied the question of finding upper bounds on the difference of eigenvalues for euclidean domains under Dirichlet condition. Many works have extended these results in the context of submanifolds. In the particular case of the sphere with a metric in the standard conformal class, we obtain new inequalities involving geometric quantities such as scalar curvature and Cheeger's constant. The method used in this construction is based on Hersch's theorem and some type of Sobolev inequalities.

ALEXANDRE GIROUARD, Université Laval
Metric upper bounds for Steklov and Laplace eigenvalues

I will discuss two upper bounds for the Steklov eigenvalues of a compact Riemannian manifold with boundary. The first involves the volume of the manifold and of its boundary, as well as the distortion, packing and volume growth constants of the boundary. The second bound is in terms of the intrinsic and extrinsic diameters of the boundary, as well as its injectivity radius. By applying these bounds to cylinders over closed manifold, we also obtain new bounds for eigenvalues of the Laplace operator on closed manifolds, in the spirit of Grigor'yan-Netrusov-Yau and of Berger-Croke. For instance, on any closed Riemannian manifold M the eigenvalue $\lambda_j(M)$ is bounded above in terms of the diameter, volume and injectivity radius:

$$\lambda_j(M)\mathrm{diam}(M)^2 \leq K(n)\frac{\mathrm{Vol}(M)}{\mathrm{inj}(M)^n}j^{n+1}.$$

I will also discuss related concentration phenomena for manifolds with boundary, akin to Gromov–Milman concentration for closed manifolds.

This is joint work with Bruno Colbois (Université de Neuchâtel).

BLAKE KEELER, Dalhousie University
A logarithmic improvement in the two-point Weyl law

In this talk, we discuss the asymptotic behavior of the spectral function of the Laplace-Beltrami operator on a compact Riemannian manifold M with no conjugate points. The spectral function, denoted $\Pi_\lambda(x, y)$, is defined as the Schwartz kernel of the orthogonal projection from $L^2(M)$ onto the eigenspaces with eigenvalue at most λ^2. In the regime where (x, y) is restricted to a sufficiently small neighborhood of the diagonal in $M \times M$, we obtain a uniform logarithmic improvement in the remainder of the asymptotic expansion for Π_λ and its derivatives of all orders. This generalizes a result of Bérard which established an on-diagonal estimate for $\Pi_\lambda(x, x)$ without derivatives. Furthermore, when (x, y) avoids a compact neighborhood of the diagonal, we obtain the same logarithmic improvement in the standard upper bound for the derivatives of Π_λ itself. We also discuss an application of these results to the study of monochromatic random waves.

ANGEL D. MARTINEZ, University of Toronto
On the symmetry conjecture for eigenfunctions

In this talk we will provide a brief introduction to the eigenfunctions of the Laplace-Beltrami operator. We will then focus on the so-called symmetry conjecture and a natural variant raised by Nadirashvili and Jakobson. In particular we will present certain results (positive and negative) obtained in collaboration with F. Torres de Lizaur, and extended by a group of students during last summer at the Fields Undergraduate Summer Research Program under our supervision. Time permitting we will comment on work in progress and open questions.