Geometric analysis and general relativity
Analyse géométrique et relativité générale
(Org: **Hari Krishna Kunduri** (Memorial) and/et **Eric Woolgar** (Alberta))

ZHONGSHAN AN, University of Connecticut
Initial boundary value problem of the vacuum Einstein equations

In general relativity, spacetime metrics satisfy the Einstein equations, which are wave equations in the harmonic gauge. The Cauchy problem for the vacuum Einstein equations has been well-understood since the work of Choquet-Bruhat. For an initial data set satisfying the vacuum constraint equations, there exists a solution to the vacuum Einstein equations and it is geometrically unique in the domain of dependence of the initial surface. On contrast, the initial boundary value problem (IBVP) has been much less understood. To solve for a vacuum metric in a region with time-like boundary, one needs well-posed boundary conditions to guarantee geometric uniqueness of the solution. However, due to gauge issues occurring on the boundary, there has not been a satisfying choice of boundary conditions. In this talk I will discuss obstacles in establishing IBVP for the vacuum Einstein equations and some progress based on joint works with Michael Anderson.

ERIC BAHUAUD, Seattle University
Analytic semigroups, bounded geometry and geometric flows

Semigroup methods provide an elegant abstract framework for studying parabolic evolution equations. In this talk I will describe joint work with Guenther, Isenberg and Mazzeo providing a criterion for a geometric elliptic operator to generate an analytic semigroup on weighted Hölder spaces on manifolds with bounded geometry. I will then outline a few applications to geometric flows.

IVAN BOOTH, Memorial University
Geometry of horizon merger during a binary black hole collision

In both rigorous mathematical and numerical studies, it most common to represent the boundary of a black hole as a marginally outer trapped surface (MOTS): a geometric structure that is closely related to (and sometimes even coincides with) a minimal surface in Riemannian geometry. As a result of gravitational wave observations,there is now a great interest in binary black hole collisions. In such a collision, the two MOTSs representing the original pair of black holes merge into a single MOTS representing the final black hole. It is only in the last few years that, through a combination of analytical and numerical techniques, this process has started to be understood. This talk will review what we now know about that process and consider the many exotic MOTS that mediate the merger. In particular I will point out how many of the "mild genericity" assumptions from theorems aiming to govern that behaviour turned out to be neither mild nor generic.

DA RONG CHENG, University of Waterloo
Existence of constant mean curvature surfaces

Constant mean curvature (CMC) surfaces arise in many different contexts and are natural generalizations of minimal surfaces. A basic question is finding CMC surfaces with controlled topology in 3-manifolds. In this talk, I will describe some recent progress on this question obtained using the mapping approach, focusing on joint work with Xin Zhou (Cornell), in which we address the genus zero case, where the surface sought after is a sphere. Our main result is that in an arbitrary Riemannian 3-sphere, for almost every H there exists a branched immersed 2-sphere with constant mean curvature H. Moreover, the existence extends to all H when the ambient metric has positive Ricci curvature.

GRAHAM COX, Memorial University
Evolution and bifurcation of marginally outer trapped surfaces

Marginally outer trapped surfaces (MOTS) are a useful proxy for black hole horizons, and are an important tool in the study of binary black hole mergers. Their smooth evolution (or lack thereof) is governed by the associated stability operator L. If L is invertible, then the MOTS evolves smoothly into a marginally outer trapped tube (MOTT). When L is not invertible the problem is more delicate, and one must use tools from bifurcation theory to study the resulting evolution. A result in this direction was given by Andersson, Mars, Metzger and Simon in 2009. In this talk I will present a refinement of their result, giving sufficient conditions for a MOTS to undergo a saddle-node bifurcation, and illustrate it with simple examples. This talk represents joint work with Liam Bussey and Hari Kunduri.

GHAZAL GESHNIZJANI, University of Waterloo/Perimeter Institute
What happened before the big bang?

I will argue why we need to remain objective about the physics of the early universe and explore different scenarios. In particular, I will present a cosmological bounce model based on Cuscuton gravity that does not have any ghosts or curvature instabilities. I will then discuss if Cuscuton bounce can provide an alternative to inflation for generating near scale-invariant scalar perturbations. While a single field Cuscuton bounce generically produces a strongly blue power spectrum (for a variety of initial/boundary conditions), scale-invariant entropy modes can be generated in a spectator field kinetically coupled to the primary field. Furthermore, this solution has no singularity, nor requires an ad hoc matching condition. Tensor modes (or gravitational waves) in Cuscuton bounce are also stable but similar to most bounce models, the produced spectrum is strongly blue and unobservable.

SHARMILA GUNASEKARAN, University of Alberta
Slow decay of waves in gravitational solitons

Gravitational solitons are globally stationary, horizonless asymptotically flat spacetimes with positive energy. Typically they arise as classical solutions to the supergravity theories. In this talk, I will address their stability at the simplest level by investigating solutions to the linear wave equation in a particular soliton spacetime. I will describe a methodology, introduced by Holzegel-Smulevici to prove that massless scalar waves in this family of soliton spacetimes cannot decay faster than inverse logarithmically in time. The proof involves the construction of quasimodes which are approximate solutions to the wave equation. This slow decay can be attributed to the stable trapping of null geodesics and is suggestive of instability at the nonlinear level. This is joint work with Hari Kunduri.

DEMETRE KAZARAS, Duke University
Comparison geometry and spacetime harmonic functions

Comparison theorems are the basis for our geometric understanding of Riemannian manifolds satisfying a given curvature condition. A remarkable example is the Gromov-Lawson toric band inequality, which bounds the distance between the two sides of a Riemannian torus-cross-interval with positive scalar curvature by a sharp constant inversely proportional to the scalar curvature's minimum. We will give a new qualitative version of this and similar band-type inequalities in dimension 3 using the notion of spacetime harmonic functions, which recently played the lead role in our recent proof of the positive mass theorem. Other applications include new versions of Bonnet-Meyer's diameter theorem and Llarull's theorem which do not require a completeness assumption. This is joint work with Sven Hirsch, Marcus Khuri, and Yiyue Zhang.

HARI KRISHNA KUNDURI, Memorial University of Newfoundland and Labrador
Toric asymptotically flat gravitational instantons

I will discuss uniqueness and existence theorems for four-dimensional complete Ricci-flat, gravitational instantons with a torus symmetry that asymptotically approach flat $S^1 \times \mathbb{R}^3$. Such instantons are characterised by their rod structure, which is data that encodes the fixed point sets of the torus action. Furthermore, we establish that for every admissible rod structure there exists an instanton that is smooth up to possible conical singularities at the axes of symmetry. The proofs involve adapting the methods that are used to establish black hole uniqueness theorems.

ERIC LING, Rutgers University
Remarks on the cosmological constant appearing as an initial condition for Milne-like spacetimes

Milne-like spacetimes are a class of $k = -1$ FLRW spacetimes which admit continuous spacetime extensions through the big bang. It was previously shown that the cosmological constant appears as an initial condition for Milne-like spacetimes under suitable assumptions on the scale factor. In this talk, we generalize this result to spacetimes which share similar geometrical properties with Milne-like spacetimes but without the strong isotropy assumption associated with them.

TRISTAN OZUCH, MIT

ANNACHIARA PIUBELLO, University of Miami
Mass and Riemannian Polyhedra

We show a new formula for the ADM mass as the limit of the total mean curvature plus the total defect of dihedral angle of the boundary of large polyhedra. In the special case of coordinate cubes, we will show an integral formula relating the n-dimensional mass with a geometrical quantity that determines the (n-1)-dimensional mass. This is joint work with Pengzi Miao.