Descent Methods in Algebra, Geometry, and Topology
Méthodes de descente en algèbre, en géométrie et en topologie

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SEIDON ALSAODY, Uppsala University
Algebras with Additional Structures via Torsors and Descent

Algebras (over commutative rings) with additional structure (such as a quadratic form or a certain decomposition) occur in various contexts. In terms of algebraic groups, this often translates to the automorphism group of the algebra being a closed subgroup of, or containing as a closed subgroup, the symmetry group of this additional structure. In such cases, a detailed understanding of the relation between these two structures can be obtained by descent methods; more precisely by studying torsors over quotients of algebraic groups.

I will report on some recent results illustrating how this is used to study the relations between different structures on algebras related to different exceptional groups. As it turns out, many classical algebraic concepts (such as triality and isotopy) fit nicely into this framework.

RUI FERNANDES, University of Illinois Urbana-Champaign
Multiplicative Ehresmann connections

I will report on recent work on multiplicative Ehresmann connections for Lie groupoid submersions, as well as their infinitesimal counterparts, the so-called IM Ehresmann connections. Time permitting I will illustrate with applications to computing (rational) Dixmier-Douady classes of gerbes.

This talk is based in joint work with Ioan Marcut (Nijmegen).

MARTIN FRANKLAND, University of Regina
Modules over bialgebroids and Beck modules

In his 1967 thesis, Beck proposed a notion of module over an object in a category. This provided a natural notion of coefficient module for André-Quillen (co)homology of any algebraic structure, generalizing the original case of commutative rings. In some cases, such as groups or Lie algebras, Beck modules are encoded by a bialgebra. The comultiplication then induces a well-behaved tensor product of modules. In work in progress with Raveen Tehara, we investigate "bialgebras with many objects" as a more general framework to encore Beck modules, where the tensor product of modules is still available. We will look at examples that fit into this framework but not that of bialgebras.

ALEXANDER KOLPAKOV, Université de Neuchâtel
Space vectors forming rational angles

We classify all sets of nonzero vectors in $\mathbb{R}^3$ such that the angle formed by each pair is a rational multiple of π. The special case of four–element subsets lets us classify all tetrahedra whose dihedral angles are multiples of π, solving a 1976 problem of Conway and Jones: there are 2 one–parameter families and 59 sporadic tetrahedra, all but three of which are related to either the icosidodecahedron or the B_3 root lattice. The proof requires the solution in roots of unity of a $W(D_6)$–symmetric polynomial equation with 105 monomials (the previous record was 12 monomials only). This is a joint work with Kiran S. Kedlaya (UCSD), Bjorn Poonen (MIT), and Michael Rubinstein (University of Waterloo).

EHUD MEIR, University of Aberdeen
Descent and generic forms using symmetric monoidal categories

Let A be some algebraic structure (e.g. Hopf or lie algebra) defined over a field K of characteristic zero. Classical descent theory asks over what fields does A have a form, and what are all the possible forms over subfields of K. In this talk I will explain how to address this problem using the theory of Deligne on symmetric monoidal categories. I will construct a symmetric monoidal category C_A, defined over a subfield K_0 of K, and I will show that forms over an intermediate field K_1 are in one to one correspondence with fiber functors from C_A to Vec_{K_1}. This also leads to the construction of generic forms, that specialize to all forms of A.

ANDREW STAAL, University of Waterloo
Small Elementary Components of Hilbert Schemes of Points

I will present some recent progress in the study of Hilbert schemes $\mathrm{Hilb}^d(\mathbb{A}^n)$ of d points in affine space, and the related (local) punctual Hilbert schemes $\mathrm{Hilb}^d(\mathcal{O}_{\mathbb{A}^n,p})$ at fixed $p \in \mathbb{A}^n$. Specifically, I will discuss some results on *elementary* components of Hilbert schemes of points and tie these to a question posed by Iarrobino in the 80's: does there exist an irreducible component of the punctual Hilbert scheme $\mathrm{Hilb}^d(\mathcal{O}_{\mathbb{A}^n,p})$ of dimension less than $(n-1)(d-1)$? I will answer this question by describing a new infinite family of irreducible components satisfying this bound, when $n = 4$. A secondary family of elementary components also arises, providing further new examples of elementary components of Hilbert schemes of points, and improving our knowledge surrounding a folklore question on the existence of certain Gorenstein local Artinian rings.

This is joint work with Matt Satriano (U Waterloo).

MARTIN SZYLD, Dalhousie University
On Tannaka Recognition and Descent for Topoi

We will recall the constructions of Tannaka theory due to Deligne, involving cogebroides, Hopf algebroids, and their representations. We will show how they can be done over a monoidal category V instead of vector spaces. The basic construction is a cogebroide L from a fiber functor, which becomes a Hopf algebroid when some conditions hold, and a lifting of this fiber functor to the category of L-comodules. A "Tannaka V-recognition theorem" gives conditions under which this lifting is an equivalence.

We will then consider Joyal-Tierney's descent theorem and its application to the structure of topoi using the spatial cover. We take categories of relations on the inverse image of the spatial cover and we obtain a "tannakian fiber functor", but for which V is the category SL of sup-lattices. The Hopf algebroid L coming from this tannakian fiber functor is no other than the formal dual of the localic groupoid G constructed by Joyal-Tierney, thus the descent theorem for topoi becomes equivalent to a Tannaka SL-recognition theorem.

The results I will present can be found in [1], I will also introduce the simpler case in which L is a Hopf algebra and G is a localic group from [2]. Time permitting, I will discuss other reasons (independent of topos theory) to develop Tannaka theory over sup-lattices that I recently beacame aware of.

[1] Dubuc, Szyld, Tannaka theory over sup-lattices and descent for topoi, TAC (2016).

[2] Dubuc, Szyld, A Tannakian Context for Galois Theory, Advances in Mathematics (2013).

VESKO VALOV, Nipissing University
On homogeneity of Cantor cubes

We discuss the question of extending homeomorphism between closed subsets of the Cantor discontinuum D^τ. It is established that any homeomorphism f between two closed subsets of D^τ can be extended to an autohomeomorphism of D^τ provided f preserves the λ-interiors of the sets for every cardinal λ. This is a non-metrizable analogue of the Ryl-Nardjewski theorem stating that if X is a proper closed subset of the Cantor set $D^{\aleph_0}$ and f is a homeomorphism of X onto $f(X)$ such that $f(\mathrm{intX}) = \mathrm{int\ f(X)}$, then there exists an autohomeomorphism of $D^{\aleph_0}$ extending f.

KIRILL ZAYNULLIN, University of Ottawa
Canonical dimension and unimodular degree of a root system

We produce a short and elementary algorithm to compute an upper bound for the canonical dimension of a spit semisimple linear algebraic group. Using this algorithm we confirm previously known bounds by Karpenko and Devyatov as well as we produce new bounds (e.g. for groups of types F_4, adjoint E_6, for some semisimple groups).