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**Advances in Operator Algebras**  
**Avancées en algèbres d'opérateurs**

(Org: **Cristian Ivanescu** (MacEwan), **Zhuang Niu** (U Wyoming) and/et **Maria Grazia Viola** (Lakehead))

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**MARTIN ARGERAMI**, University of Regina

*Classification of certain finite-dimensional Operator Systems*

Operator systems are very easy to define, but surprisingly hard to classify. We will describe the classification of a certain family of operator systems generated by normal operators. This brings some clarity on previously not-so-well-understood cases, and it gives us ideas to attack some old problems.

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**JULIAN BUCK**, Okanagan College

*The structure of crossed products by automorphisms of  $C(X, D)$*

We construct centrally large subalgebras in crossed products of the form  $C^*(\mathbb{Z}, C(X, D), \alpha)$  in which  $D$  is simple,  $X$  is compact metrizable,  $\alpha$  induces a minimal homeomorphism  $h : X \rightarrow X$ , and a mild technical assumption holds. We use this construction to prove structural properties of the crossed product, such as (tracial)  $\mathcal{Z}$ -stability, stable rank one, real rank zero, and pure infiniteness, in a number of examples. Our examples are not accessible via methods based on finite Rokhlin dimension, either because  $D$  is not  $\mathcal{Z}$ -stable or because  $X$  is infinite dimensional. This is joint work with Dawn Archey and N. Christopher Phillips.

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**ANDREW DEAN**, Lakehead University

*Structure and classification for real  $C^*$ -algebras*

The program to classify certain simple  $C^*$ -algebras has now reached a very advanced state, for complex  $C^*$ -algebras. The situation for real  $C^*$ -algebras is much less clear. We will give a survey of structure and classification results for real  $C^*$ -algebras.

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**XUANLONG FU**, University of Toronto

*Tracial matricial structure and stable rank one*

Let  $A$  be a separable (not necessarily unital) simple  $C^*$ -algebra with strict comparison. We show that if  $A$  has property (TM) then  $A$  has stable rank one and the canonical map  $\Gamma$  from the Cuntz semigroup of  $A$  to the corresponding lower-semicontinuous affine function space is surjective. The converse also holds.

As a by-product, we find that a separable simple  $C^*$ -algebra, which has almost stable rank one must have stable rank one, provided it has strict comparison and the canonical map  $\Gamma$  is surjective.

This is a joint work with Huaxin Lin.

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**DAVID KRIBS**, University of Guelph

*Multiplicative domains, complementary quantum channels, and entanglement breaking rank*

Quantum entanglement can be studied through the theory of completely positive maps in a number of ways, including by making use of the Choi-Jamolkowski isomorphism, which identifies separable states with entanglement breaking quantum channels, and optimal ensemble length with entanglement breaking rank. The multiplicative domain is an important operator structure in the theory of completely positive maps. In this talk, I'll discuss my recent work with collaborators in which we introduce a new technique to determine if a channel is entanglement breaking and to evaluate entanglement breaking rank, based on an analysis of multiplicative domains determined by complementary quantum channels. We give a full description of the class of entanglement breaking channels that have a projection as their Choi matrix (which turns out to be rather restrictive), and

we show the entanglement breaking and Choi ranks of such channels are equal. This talk is based on joint work with Jeremy Levick, Rajesh Pereira and Miza Rahaman.

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**DAN KUCEROVSKY**, Univ. of New Brunswick at Fredericton  
*Comparing Hilbert module unitaries with Hilbert space unitaries; and multiplicative unitaries.*

We show that a  $A$ -linear map of Hilbert  $A$ -modules is induced by a unitary Hilbert module operator if and only if it extends to an ordinary unitary on appropriately defined enveloping Hilbert spaces. Applications to the theory of multiplicative unitaries compute the equivalence classes of Hilbert modules over a class of  $C^*$ -algebraic quantum groups. We develop a theory that for example could be used to show non-existence of certain co-actions.

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**BOYU LI**, University of Waterloo  
*Imprimitivity theorems arising from Zappa-Szep actions on Fell bundles*

Imprimitivity theorems, such as Green's imprimitivity theorem, establish the Morita equivalence between  $C^*$ -algebras arising from certain group actions. Recently, we introduced the notion of Zappa-Szep product of Fell bundles by groupoids that encodes a two-way action between the operator algebra and the groupoid. This in turn leads to a more general imprimitivity theorem arising from these dynamics. This is a joint work with Anna Duwenig.

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**YUQING FRANK LIN**, Texas A&M University  
*A multiplicative ergodic theorem for von Neumann algebra valued cocycles*

Oseledets' multiplicative ergodic theorem (MET), an important tool in smooth ergodic theory, may be viewed as a dynamical version of the Schur decomposition from linear algebra. Past generalizations of the MET to infinite dimensional spaces have assumed quasi-compactness conditions on the operators. Without assuming any compactness conditions, we use a geometric result of Karlsson-Margulis to obtain an MET with operators in von Neumann algebras with semi-finite trace.

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**ANAMARIA SAVU**, University of Alberta  
*Processes with zero-range interaction and integrability*

Zero-range processes are interacting particle systems where particles hop between the lattice sites with rates that depend solely on the number of particles of the departure site. The behaviour on the long wavelength and time-scale of zero-range processes have been extensively studied, and asymptotic results such as hydrodynamic scaling limit, central limit theorem, or large deviations of the empirical distribution of particles have been established. A specific zero-range process on the 1-dimensional infinite lattice, the  $q$ -Boson system, was introduced by Sasamoto and Wadati. The  $q$ -Boson specifies that a single particle leaves a site at a rate equal to  $[n]$ , the  $q$ -integer of the site occupancy  $n$ . Notably, the  $q$ -Boson was shown to be integrable in the sense that a class of eigenfunctions can be constructed for the Hamiltonian of the process. Later the  $q$ -Boson system was generalized by Takeyama using the algebra structure generated by the multiplication and divided-difference operators of Lascoux and Schützenberger. The system built by Takeyama is totally asymmetric, exhibits zero-range interaction, and has the novel feature that any number of available particles can leave the site. We show that the system of Takeyama can be enhanced to allow the movement of particles to both left and right and remain integrable. Also, we discuss the attractiveness and propose that the hydrodynamic scaling limit of the system is a first-order quasilinear partial differential equation.

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**PAUL SKOUFRANIS**, York University  
*Non-Commutative Stochastic Processes and Bi-Free Probability*

When Voiculescu introduced the notion of bi-free independence, he envisioned that the theory would model the transition between past and future states. As such, the objects under consideration were called "pairs of faces" after the Roman god of transitions Janus who is often depicted with two faces, one looking to the past and one to the future. In this talk, we will

discuss how the transition operators for non-commutative stochastic processes can be modelled using technology from bi-free probability. Several important examples are recovered with this approach and new formula are obtained for processes with free increments. The benefits of this approach are also discussed.

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**NICO SPRONK**, University of Waterloo

*Amenability from operator algebras to algebras of harmonic analysis*

I will give a brief overview of amenability theory in the sense of B. Johnson as it applies to operator algebras, and indicate how it effects certain algebras of harmonic analysis. I will highlight recent work of mine around Fourier-Stieltjes algebras.

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**ANDREW TOMS**, Purdue University

*The homotopy type of Cuntz classes in real rank zero C\*-algebras*

Roughly speaking, the Cuntz semigroup generalizes the Murray-von Neumann semigroup for projections to the broader setting of positive elements. Introduced by Cuntz in 1978, it has enjoyed a resurgence since the mid 2000s and has become an important tool in the structure and classification theory of nuclear separable C\*-algebras. In this talk we examine the homotopy type of Cuntz classes in real rank zero C\*-algebras, and identify their homotopy groups completely for a class that includes AF and irrational rotation algebras.

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**ELISABETH WERNER**, Case Western Reserve University

*On the geometry of projective tensor products*

We study the volume ratio of the projective tensor products  $\ell_p^n \otimes_\pi \ell_q^n \otimes_\pi \ell_r^n$  with  $1 \leq p \leq q \leq r \leq \infty$ . We obtain asymptotic formulas that are sharp in almost all cases. As a consequence of our estimates, these spaces allow a nearly Euclidean decomposition of Kashin type whenever  $1 \leq p \leq q \leq r \leq 2$  or  $1 \leq p \leq 2 \leq r \leq \infty$  and  $q = 2$ . Also, from the Bourgain-Milman bound on the volume ratio of Banach spaces in terms of their cotype 2 constant, we obtain information on the cotype of these 3-fold projective tensor products. Our results naturally generalize to  $k$ -fold products  $\ell_{p_1}^n \otimes_\pi \cdots \otimes_\pi \ell_{p_k}^n$  with  $k \in \mathbb{N}$  and  $1 \leq p_1 \leq \cdots \leq p_k \leq \infty$ .

Based on joint work with O. Giladi, J. Prochno, C. Schuett and N. Tomczak-Jaegermann.