
WANLIN LI, CRM

On the vanishing of twisted L -functions of elliptic curves over function fields

We investigate the vanishing at $s = 1$ of the twisted L -functions of elliptic curves E defined over the rational function field $\mathbb{F}_q(t)$, for twists by Dirichlet characters of prime order $\ell \geq 3$, from both a theoretical and numerical point of view. In the case of number fields, it is predicted that such vanishing is a very rare event, and our numerical data seems to indicate that this is also the case over function fields for non-constant curves. For constant curves, we prove that if there is one χ_0 such that $L(E, \chi_0, 1) = 0$, then there are infinitely many. Finally, we provide some examples which show that twisted L -functions of constant elliptic curves over $\mathbb{F}_q(t)$ behave differently than the general ones. This is joint work with Antoine Comeau-Lapointe, Chantal David, and Matilde Lalin.