DYLAN LANGHARST, Kent State University

Measure Theoretic Minkowski's Existence Theorem and Projection Bodies

The Brunn-Minkowski Theory has seen several generalizations over the past century. Many of the core ideas have been generalized to measures. With the goal of framing these generalizations as a measure theoretic Brunn-Minkowski theory, we prove the Minkowski existence theorem for a large class of Borel measures with density, denoted by Λ': for ν a finite, even Borel measure on the unit sphere and $\mu \in \Lambda'$, there exists a symmetric convex body K such that

$$d\nu(u) = c_{\mu,K} dS_{\mu,K}(u),$$

where $c_{\mu,K}$ is a quantity that depends on μ and K and $dS_{\mu,K}(u)$ is the surface area-measure of K with respect to μ. Examples of measures in Λ' are homogeneous measures (with $c_{\mu,K} = 1$) and probability measures with continuous densities (e.g. the Gaussian measure). We will also consider measure dependent projection bodies $\Pi_\mu K$ by classifying them and studying the isomorphic Shephard problem: if μ and ν are even, homogeneous measures with density and K and L are symmetric convex bodies such that $\Pi_\mu K \subset \Pi_\nu L$, then can one find an optimal quantity $\mathcal{A} > 0$ such that $\mu(K) \leq \mathcal{A}\nu(L)$? Among other things, we show that, in the case where $\mu = \nu$ and L is a projection body, $\mathcal{A} = 1$.