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Jacobi-Toda systems and interfaces with higher multiplicities

We consider solutions to Allen-Cahn equation

$$\epsilon^2 \Delta u + u - u^3 = 0$$

with bounded energy

$$E_\epsilon(u) = \frac{\epsilon}{2} \int |\nabla u|^2 + \frac{1}{4\epsilon} \int (1-u^2)^2 \leq C$$

It is known that the zero level sets converges $\sum_j m_j \Gamma_j$, where each Γ_j is a minimal surface. When $m_j = 1$, the minimal surface is simple and one can show $C^{1,\alpha}$ or $C^{2,\alpha}$ convergence. In this talk we will study the most difficult case, i.e., $m_j \geq 2$, higher multiplicities. Chosh-Matoulidis (Annals of Math 2019) showed that when $m_j = 2, 3 \leq d \leq 7$, there must exist a nontrivial positive Jacobi field. We first derive a second order condition for interfaces with higher multiplicities, and use it to construct solutions with multiplicity 2. Then we give a complete classification of geodesic nets with higher multiplicity in the two-dimensional case. We construct and compute the precise Morse index in terms of m_j and the number of intersections. The key role played in the above analysis is the behavior of solutions to Jacob-Toda systems. (Joint work with F. Pacard and Y. Liu.)