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Optimal Transport on the Sphere

Recently, much progress has been made using wide-stencil finite-difference schemes to provide convergence frameworks for optimal transport and related Monge-Ampère-type PDE problems. One particular merit of these methods lies in their ability to deal with viscosity solutions, as first hinted at in a 1991 paper by Barles-Souganidis. However, these methods have so far not been adapted to geometries other than subsets of Euclidean space. Recent applications including global moving-mesh methods in meteorology and the reflector antenna problem have inspired work on the optimal transport problems on the sphere, which utilize the squared geodesic cost and a negative logarithmic cost, respectively. Here, we construct a numerical convergence framework for optimal transport on the sphere with both cost functions in mind. We show that a wide-stencil finite-difference scheme can be constructed to solve the optimal transport problem on the sphere that has guaranteed convergence to a modified PDE in both smooth and non-smooth cases, where in the latter case we must construct underestimating schemes. We supplement this theory with a particular construction of the scheme which satisfies the hypotheses of the convergence theorem and demonstrate its effectiveness at computing heretofore unsolved problems on the sphere.