DANIEL LE, Purdue University

A mod p local-global compatibility result for generic Fontaine-Laffaille representations

By work of Khare-Wintenberger, Colmez, Emerton, and others, the commuting actions of $\mathrm{GL}_2(\mathbb{A})$ and $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ on the $\overline{\mathbf{F}}_p$-cohomology of the tower of modular curves realizes a mod p Langlands correspondence, characterized by the Eichler-Shimura relation at good primes and Colmez's Montreal functor at p. With no conjectural formulation of a mod p Langlands correspondence for $\mathrm{GL}_n(\mathbb{Q}_p)$ at present, it is natural to ask if a local $\mathrm{Gal}(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$-representation can be recovered from the corresponding $\mathrm{GL}_n(\mathbb{Q}_p)$-representation appearing in the cohomology of an appropriate adelic quotient. We give an affirmative answer in some generic Fontaine-Laffaille cases (also allowing unramified extensions of $\mathbb{Q}_p$). This is joint work with Le Hung, Morra, Park, and Qian.