Quantum Mathematics
Mathématiques quantiques

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MAISSAM BARKESHLI, University of Maryland
Anomalies in (2+1)D fermionic topological phases and (3+1)D state sums for fermionic SPTs

I will describe a way to compute anomalies in general (2+1)D fermionic topological phases. First, a mathematical characterization of symmetry fractionalization for (2+1)D fermionic topological phases is presented, and then this data will be used to define a (3+1)D state sum for a topologically invariant path integral that depends on a generalized spin structure and G bundle on a 4-manifold. This path integral is a cobordism invariant and describes a (3+1)D fermion symmetry-protected topological state (SPT). The special case of time-reversal symmetry with $T^2 = -1^F$ gives a $\mathbb{Z}_{16}$ invariant of the 4D Pin$^+$ smooth bordism group, and gives an example of a state sum that can distinguish exotic smooth structure.

MENG CHENG, Yale University
Fractionalization and anomaly in symmetry-enriched topological phases

I will discuss recent results in the theory of symmetry-enriched topological phases, with a focus on the (2+1) case. I will review the classification of symmetry-enriched topological order and present general formula to compute relative 't Hooft anomaly for bosonic topological phases. I will also discuss partial results for fermionic topological phases and open questions.

THEO JOHNSON-FREYD, Dalhousie University and Perimeter Institute
Classification of topological orders

Topological orders have a mathematical axiomatization in terms of their higher fusion categories of extended operators; the characterizing property of these higher fusion categories is that they are satisfy a nondegeneracy condition. After overviewing some of the higher category theory that goes into this axiomatization, I will describe what we do and don't know about the classification of topological orders in various dimensions.

DAVID KRIBS, University of Guelph
Operator theory and distinguishing quantum states with LOCC

In this talk, I'll discuss my work with collaborators on a fundamental topic in quantum information: Given a known set of quantum states, when can two parties distinguish the states under the restricted and hybrid classical-quantum communication setting called local (quantum) operations and classical communication (LOCC). I'll show how we've been able to make use of some tools from operator theory and operator algebras to develop techniques that solve certain subproblems, and briefly discuss our other ongoing and related work. This talk is based on joint works with Comfort Mintah, Michael Nathanson, and Rajesh Pereira.

PETER KRISTEL, University of Manitoba
Connes fusion of the free fermions on the circle

A conformal net on S^1 is an assignment $\mathcal{A} : \{\text{open subsets of } S^1\} \to \{\text{von Neumann algebras acting on } \mathcal{F}\}$, which satisfies a slew of axioms motivated by quantum field theory. In this talk, I will consider the free fermionic conformal net. In this case, the Hilbert space $\mathcal{F}$ is the Fock space generated by the positive energy modes of square-integrable spinors on the circle $L^2(S^1, \mathbb{S})$; and the von Neumann algebras are Clifford algebras generated by those elements of $L^2(S^1, \mathbb{S})$ whose support lies in $I \subset S^1$. After going over this construction, I will argue that given an open interval $I \subset S^1$, one can equip $\mathcal{F}$ with the structure of $\mathcal{A}(I)$-$\mathcal{A}(I)$-bimodule. I will then outline the construction of a canonical isomorphism of bimodules $\mathcal{F} \boxtimes_{\mathcal{A}(I_-)} \mathcal{F} \to \mathcal{F}$, where $\boxtimes_{\mathcal{A}(I_-)}$ stands for the Connes fusion product over the algebra assigned to the lower semi-circle I_-. If time permits, I will discuss some (anticipated) applications of this isomorphism, for example in string geometry, or in the construction of the free fermion *extended* topological field theory.

SÉBASTIEN LORD, University of Ottawa
Secure Software Leasing Without Assumptions

Quantum cryptography is known for enabling functionalities that are unattainable using classical information alone. Recently, Secure Software Leasing (SSL) has emerged as one of these areas of interest. Given a circuit C from a circuit class, SSL produces an encoding of C that enables a recipient to evaluate C and also enables the originator of the software to later verify that the software has been returned, meaning that the recipient has relinquished the possibility to further use the software. Such a functionality is unachievable using classical information alone, since it is impossible to prevent a user from keeping a copy of the software. Recent results have shown the achievability of SSL using quantum information for compute-and-compare functions (a generalization of point functions). However, these prior works all make use of setup or computational assumptions. We show that SSL is achievable for compute-and-compare circuits without any assumptions.

We proceed by studying quantum copy-protection, which is a notion related to SSL, but where the encoding procedure inherently prevents a would-be quantum software pirate from splitting a single copy of an encoding for C into two parts each allowing a user to evaluate C. Using quantum message authentication codes, we show that point functions can be copy-protected without any assumptions against one honest and one malicious evaluator. We then show that a generic honest-malicious copy-protection scheme implies SSL. By prior work, this yields SSL for compute-and-compare functions.

This is joint work with Anne Broadbent, Stacey Jeffery, Supartha Podder, and Aarthi Sundaram.

JOSEPH MACIEJKO, University of Alberta
Hyperbolic band theory

The notions of Bloch wave, crystal momentum, and energy bands are commonly regarded as unique features of crystalline materials with commutative translation symmetries. Motivated by the recent realization of hyperbolic lattices in circuit QED, I will present a hyperbolic generalization of Bloch theory, based on ideas from Riemann surface theory and algebraic geometry. The theory is formulated despite the non-Euclidean nature of the problem and concomitant absence of commutative translation symmetries. The general theory will be illustrated by examples of explicit computations of hyperbolic Bloch wavefunctions and bandstructures.

CIHAN OKAY, Bilkent University
A hidden variable model for universal quantum computation with magic states on qubits

A central question in quantum information theory is to determine physical resources required for quantum computational speedup. In the model of quantum computation with magic states classical simulation algorithms based on quasi-probability distributions, such as discrete Wigner functions, are used to study this question. For quantum systems of odd local dimension it has been known that negativity in the Wigner function can be seen as a computational resource. The case of qubits, however, resisted a similar approach for some time since the nice properties of Wigner functions for odd dimensional systems no longer hold for qubits. In our recent work we construct a hidden variable model, which replaces the Wigner function representation, for qubit systems where any quantum state can be represented by a probability distribution over a finite state space and quantum operations correspond to Bayesian update of the probability distribution. When applied to the model of quantum computation with magic states the size of the state space only depends on the number of magic states. This is joint work with Michael Zurel and Robert Raussendorf; Phys. Rev. Lett. 125, 260404 (2020).

ARTUR SOWA, University of Saskatchewan
Quantum applications of harmonic analysis on the group of positive rationals

Harmonic analysis on the multiplicative group of positive rational numbers ($\mathbb{Q}_+$) has not been part of the common quantum-theoretic toolkit. In this talk, I will discuss how it lends itself to the analysis of operators in $\ell_2(\mathbb{N})$, in some cases leading to spectacular new insights into their spectral properties. I will also discuss its application in a study of the Bose-Hubbard model, i.e. a model of an array of bosons with the nearest-neighbour interactions. The Fourier transform on $\mathbb{Q}_+$ uncovers the model's unobvious symmetries and surprising connections with other structures. In addition, I will report a rigorous, albeit computer-assisted, proof of the existence of quantum phase transitions in finite quantum systems of this type. The study of the Bose-Hubbard model has been carried out in collaboration with Prof. Jonas Fransson (Department of Physics and Astronomy, University of Uppsala).

KAORI TANAKA, University of Saskatchewan
Topological superconductivity in quasicrystals

Majorana fermions – charge-neutral spin-1/2 particles that are their own antiparticles – have been detected in one- and two-dimensional topological superconductors. Due to the non-Abelian exchange statistics that they obey, Majorana fermions open the door to new and powerful methods of quantum information processing. Motivated by the recent experimental discovery of superconductivity in a quasicrystal, we study the possible occurrence of non-Abelian topological superconductivity (TSC) in two-dimensional quasicrystals by the same mechanism as in crystalline counterparts. We show that the TSC phase can be realised in Penrose and Ammann-Beenker quasicrystals, where the Bott index is unity. Furthermore, we confirm the existence of Majorana zero modes along the surfaces and in a vortex at the centre of the system, consistently with the bulk-boundary correspondence.

LUC VINET, C R M
Entanglement of Free Fermions on Graphs

The entanglement of free fermions on Hamming graphs will be discussed. This will be used to showcase how tools of algebraic combinatorics such as the Terwilliger algebra are well suited for this analysis. The usefulness of a Heun operator generalization will also be stressed and extensions to other association schemes will be mentioned.

CHONG WANG, Perimeter Institute
Stiefel liquids: possible non-Lagrangian quantum criticality from intertwined orders

We propose a new type of critical quantum liquids, dubbed Stiefel liquids, based on 2+1 dimensional Wess-Zumino-Witten models on target space SO(N)/SO(4). We show that the well known deconfined quantum critical point and U(1) Dirac spin liquid are unified as two special examples of Stiefel liquids, with N=5 and N=6, respectively. Furthermore, we conjecture that Stiefel liquids with $N > 6$ are non-Lagrangian, in the sense that the theories do not (at least not easily) admit any weakly-coupled UV completion. Such non-Lagrangian states are beyond the paradigm of parton gauge theory familiar in the study of exotic quantum liquids in condensed matter physics. The intrinsic absence of mean-field construction also makes it difficult to decide whether a non-Lagrangian state can emerge from a specific UV system (such as a lattice spin system). For this purpose we hypothesize that a quantum state is emergible from a lattice system if its quantum anomalies match with the constraints from the (generalized) Lieb-Schultz-Mattis theorems. Based on this hypothesis, we find that some of the non-Lagrangian Stiefel liquids can indeed be realized in frustrated quantum spin systems, for example, on triangular or Kagome lattice, through the intertwinement between non-coplanar magnetic orders and valence-bond-solid orders.

JUVEN WANG, Harvard University
Ultra Unification: Quantum Fields Beyond the Standard Model

Strong, electromagnetic, and weak forces were unified in the Standard Model (SM) with spontaneous gauge symmetry breaking. These forces were further conjectured to be unified in a simple Lie group gauge interaction in the Grand Unification (GUT). Here I propose a theory beyond the SM and GUT by adding new gapped Topological Phase Sectors consistent with the nonperturbative global anomaly cancellation and cobordism constraints (especially from the baryon minus lepton number B – L, the electroweak hypercharge Y, and the mixed gauge-gravitational anomaly). Gapped Topological Phase Sectors are constructed via symmetry extension, whose low energy contains unitary Lorentz invariant topological quantum field theories (TQFTs): either 3+1d non-invertible TQFT (long-range entangled gapped phase), or 4+1d invertible or non-invertible TQFT (short-range or long-range entangled gapped phase). Alternatively, there could also be right-handed neutrinos, or gapless unparticle conformal field theories, or their combinations to altogether cancel the anomaly. We propose that a new high-energy physics frontier beyond the conventional 0d particle physics relies on the new Topological Force and Topological Matter including gapped extended objects (gapped 1d line and 2d surface operators or defects, etc., whose open ends carry deconfined fractionalized particle or anyonic string excitations). Physical characterizations of these gapped extended objects require the mathematical theories of cohomology, cobordism, or category. I will also fill in the dictionary between math, QFT, and condensed/quantum matter terminology, and elaborate on the global anomalies of Z2, Z4, Z16 classes useful for beyond SM. Work is based on https://doi.org/10.1103/PhysRevD.103.105024, arXiv:2012.15860, arXiv:2008.06499, arXiv:2006.16996, arXiv:1910.14668.

JINGLEI ZHANG, Institute for Quantum Computing
SU(2) hadrons on a quantum computer

Lattice gauge theories are relevant in many fields of physics, and simulations with quantum computers can become a powerful tool to study them, especially in regimes inaccessible to classical numerical methods. In particular, non-Abelian gauge theories, which among other things describe fundamental particles' interactions, are of great interest. In this talk I will discuss the first quantum simulation of a non-Abelian lattice gauge theory that includes dynamical matter. I will show how the theory is formulated in order to include colour degrees of freedom, and how this allows for the existence of baryons in the model, which do not exist in Abelian theories. A quantum computation of the low-lying spectrum of the model is performed on an IBM superconducting platform using a variational quantum eigensolver. This proof-of-concept demonstration was made possible by a resource-efficient approach in the design of the quantum algorithm, and lays out the foundation for further development of the field. This talk is based on arXiv:2102.08920.