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*The Brown measures of free circular and multiplicative Brownian motions with nontrivial initial conditions*

Free probability theory is a noncommutative version of probability theory based on free independence. Free random variables can be viewed as limit objects of certain random matrices. The limits of matrix-valued Brownian motions are described as various Brownian motions in the framework of free probability.

I will discuss briefly a joint work with CW Ho on the Brown measures of free Brownian motions with certain nontrivial initial conditions, where the subordination functions played a key role. The famous circular law (due to Ginibre, Girko, Bai, Tao, Vu and many others) states that  $n$  by  $n$  square random matrices with independent and i.i.d. entries that have mean zero and variance  $1/n$  converges to the uniform distribution on the unit disk as the size  $n$  tends to infinity. Our result for circular Brownian motion provides a density formula for the candidate of the limit operator of those random matrices perturbed by any deterministic Hermitian matrices that converge to some limit. The multiplicative version of this result extends a recent result of Driver-Hall-Kemp where they used some novel PDE methods to calculate the Brown measure of the free multiplicative Brownian motion with identity as the initial condition.