Low dimensional topology
Topologie en basses dimensions
(Org: **Idrissa Ba** and/et **Adam Clay** (Manitoba))

HANS BODEN, McMaster University
The Gordon-Litherland pairing for knots in thickened surfaces

We introduce the Gordon-Litherland (GL) pairing for knots and links in thickened surfaces that bound unoriented spanning surfaces. Using the GL pairing, we define several invariants (signature, determinant, and nullity) and discuss how to compute them from the Tait graph and Goeritz matrix associated to a checkerboard surface. The invariants depend very weakly on the choice of spanning surface, and in fact only on its S^*-equivalence class.

We will discuss several applications to problems such as detecting the minimal supporting genus, determining sliceness, and slice genus of virtual knots. The GL pairing can also be regarded as the relative intersection pairing on a 4-manifold obtained as the 2-fold cover along the surface. This talk represents joint work in progress with M. Chrisman and H. Karimi.

TYRONE GHASWALA, Université du Québec à Montréal
Boundary Dehn twists are excellent

Suppose you have a compact orientable surface with one boundary component. It is known that the Dehn twist about a curve isotopic to the boundary component is not quite like all the other Dehn twists. For one thing, it is central in the mapping class group of the surface. I will prove that such Dehn twists are co-final in every left-ordering of the mapping class group, making them even cooler than originally thought! I will then discuss what this tells us about mapping class group actions on the real line and the fractional Dehn twist coefficient.

This is work in progress with Adam Clay.

YING HU, University of Nebraska Omaha
Slope detection and toroidal 3-manifolds

The L-space Conjecture says that for a prime 3-manifold, properties NLS (not being an L-space), LO (having left-orderable fundamental group), and CTF (admitting a co-orientable taut foliation), are equivalent.

We investigate these properties for toroidal 3-manifolds through the notion of slope detection. We show that all toroidal integer homology spheres are LO, and that the n-fold cyclic branched covers of a prime satellite knot are NLS and LO, and are CTF if its companion is fibered. We also prove a partial extension of the latter result to links and confirm a folklore conjecture that prime satellite links are never quasi-alternating.

This is joint work with Steve Boyer and Cameron Gordon.

KASIA JANKIEWICZ, University of Chicago
Boundary rigidity for groups acting on product of trees

The visual boundary is a well-defined compactification of a hyperbolic or CAT(0) space. For hyperbolic groups the boundary is unique up to homeomorphism. However, Croke-Kleiner constructed examples of CAT(0) groups acting geometrically on CAT(0) spaces with non-homeomorphic boundaries. I will discuss the question of the uniqueness of the boundary for groups acting geometrically on product of two trees. This is a wide family of groups including product of free groups, as well as some simple groups. This is joint work with Annette Karrer, Kim Ruane and Bakul Sathaye.

JONATHAN JOHNSON, University of Texas at Austin
Bi-Orderability and Branched L-Space Knots

The orderability of 3-manifold groups has become a topic of interest in the last couple of decades. For example, the L-space conjecture posits a surprising relationship between the left-orderability of the fundamental group of a rational homology sphere, the foliations of that manifold, and the manifold's Heegaard Floer homology. However, the fundamental groups of 3-manifolds with positive first Betti number are always left-orderable. In particular, knot groups are left-orderable. In this situation, bi-orderability is more illuminating. In this talk, I will discuss a couple of results on the bi-orderability of knot groups, and how these results are related to properties of the cyclic branched covers of a knot.

HOMAYUN KARIMI, McMaster University
A characterization of alternating links in thickened surfaces

We use an extension of Gordon-Litherland pairing to thickened surfaces to give a topological characterization of alternating links in thickened surfaces. If Σ is a closed oriented surface and F is a compact unoriented surface in $\Sigma \times I$, then the Gordon-Litherland pairing defines a symmetric bilinear pairing on the first homology of F. A compact surface in $\Sigma \times I$ is called definite if its Gordon-Litherland pairing is a definite form. We prove that a non-split link L in a thickened surface is alternating if and only if it bounds two definite surfaces of opposite sign. This is joint work with Hans U. Boden.

HEEJOUNG KIM, University of Illinois at Urbana-Champaign
End-periodic homeomorphisms and volumes of mapping tori

The Mapping class group $Map(S)$ of a finite type surface S has been studied and generally well understood. In particular, there is the Nielsen-Thurston classification of elements of $Map(S)$. One of the types of elements is called a pseudo-Anosov homeomorphism, which can be characterized by the hyperbolicity of the manifold associated with an element called a mapping torus. For a pseudo-Anosov element $f \in Map(S)$, Brock and Agol gave the upper bound for the volume of the mapping torus of f. Motivated by their results, we consider an end-periodic homeomorphism f for a certain infinite type surface S and the volume $V(f)$ of the convex core of the mapping torus of f. We give an upper bound on $V(f)$ in terms of the asymptotic translation length of f on the pants graph. This is a joint work with Elizabeth Field, Christopher Leininger, and Marissa Loving.

SIDDHI KRISHNA, Georgia Institute of Technology
Taut foliations, Dehn surgery, and braid positivity

The L-space conjecture predicts a surprising relationship between the algebraic, geometric, and Floer-homological properties of a 3-manifold Y. In particular, it predicts exactly which 3-manifolds admit a "taut foliation". In this talk, I'll discuss some of my past and forthcoming work investigating these connections, with a view towards "braid positive knots" (i.e. the knots realized as the closure of positive braids). I'll focus on applications: in particular, I'll present some new obstructions to braid positivity, and a new unknot detector. No background in foliations or Floer homology theories will be assumed. All are welcome!

MARISSA LOVING, Georgia Institute of Technology-Main Campus
Covers, Curves, and Length Spectra

I will share some of my ongoing work with Tarik Aougab, Max Lahn, and Nick Miller in which we explore the simple length spectrum rigidity of hyperbolic metrics arising from Sunada's construction. Along the way we give a characterization of equivalent covers (not necessarily regular) in terms of simple elevations of curves, generalizing previous work with Aougab, Lahn, and Xiao.

KATHRYN MANN, Cornell University
Homeomorphisms of surfaces and the fine curve graph

In recent work, Bowden, Hensel and Webb studied a variant of the classical curve graph, whose vertices are all simple closed curves (not the classical version with curves up to isotopy!) In my talk I'll explain why this is a useful, object and describe new joint work with the aforementioned BHW and Emmanuel Militon, that relates the dynamics of isotopically trivial homeomorphisms of the surface to the dynamics of their induced actions on this big curve graph, giving us new tools to study groups acting on surfaces.

DUNCAN MCCOY, UQAM
Smoothing singularities vs definite fillings

There is a conjecture by Kollár that asserts that a large family of rational surface singularities admit a unique smoothing. Topologically speaking, a smoothing of a rational surface singularity corresponds to finding a negative definite 4-manifold filling the link of the singularity. It is natural, therefore, to wonder if one can establish a topological analogue of Kollár's conjecture by studying the negative definite manifolds fillings of these link singularities. I will discuss some joint work with Paolo Aceto and JungHwan Park relating to this question. No knowledge of singularity theory will be assumed.

INA PETKOVA, Dartmouth College
Annular link Floer homology and $\mathfrak{gl}_{1|1}$

The Reshetikhin-Turaev construction for the quantum group $U_q(\mathfrak{gl}_{1|1})$ sends tangles to $\mathbb{C}(q)$-linear maps in such a way that a knot is sent to its Alexander polynomial. Tangle Floer homology is a combinatorial generalization of knot Floer homology which sends tangles to (homotopy equivalence classes of) bigraded dg bimodules. In earlier work with Ellis and Vertesi, we show that tangle Floer homology categorifies a Reshetikhin-Turaev invariant arising naturally in the representation theory of $U_q(\mathfrak{gl}_{1|1})$; we further construct bimodules $\mathcal{E}$ and $\mathcal{F}$ corresponding to E, F in $U_q(\mathfrak{gl}_{1|1})$ that satisfy appropriate categorified relations. After a brief summary of this earlier work, I will discuss how the horizontal trace of the $\mathcal{E}$ and $\mathcal{F}$ actions on tangle Floer homology gives a $\mathfrak{gl}_{1|1}$ action on annular link Floer homology that has an interpretation as a count of certain holomorphic curves. This is based on joint work in progress with Andy Manion and Mike Wong.

WILL RUSHWORTH, McMaster
An application of link parity

Joint work with Hans Boden. Let D be a link diagram on an orientable surface Σ. A *parity* is a designation of the crossings of D as either *even* or *odd*, satisfying certain axioms.

Parity is a very useful tool in the study of knots in 3-manifolds of the form $\Sigma \times I$ (and related theories), but extending such methods to links of more than one component has proven to be difficult. We describe a new parity for a class of links in $\Sigma \times I$, and use it to prove a minimality result for link diagrams, generalizing a result of Manturov in the case of knot diagrams.

ANH TRAN, University of Texas at Dallas
Classical pretzel knots and left-orderability

Heegaard Floer homology is a package of 3-manifold invariants introduced by Ozsvath and Szabo. Manifolds with minimal Heegaard Floer homology are called L-spaces. The L-space conjecture of Boyer, Gordon and Watson states that an irreducible rational homology 3-sphere is an L-space if and only if its fundamental group is not left-orderable. In this talk, we will discuss this conjecture for 3-manifolds obtained from the classical pretzel knots by Dehn surgeries.

HANNAH TURNER, University of Texas at Austin
Branched cyclic covers and L-spaces

A 3-manifold is called an L-space if its Heegaard Floer homology is "simple." No characterization of all such "simple" 3-manifolds is known. Manifolds obtained as the double-branched cyclic cover of a knot in the 3-sphere give many examples of L-spaces. In this talk, I'll discuss the search for L-spaces among higher index branched cyclic covers of knots. In particular, I'll give new examples of knots whose branched cyclic covers are L-spaces for every index n. This is joint work with Ahmad Issa.

YVON VERBERNE, Georgia Institute of Technology
The asymptotic dimension of big mapping class groups

In 2010, Bestvina-Bromberg-Fujiwara proved that the mapping class group of a finite-type surface has finite asymptotic dimension. In contrast, we will show the mapping class group of an infinite-type surface has infinite asymptotic dimension if it contains an essential shift. This work is joint with Curtis Grant and Kasra Rafi.

BIJI WONG, Max Planck Institute for Mathematics
d-invariants of double branched covers of links

Using Heegaard Floer homology, one can associate to a rational homology 3-sphere Y, equipped with a spin^c-structure $\mathfrak{s}$, a rational number, commonly referred to as the d-invariant of $(Y, \mathfrak{s})$. d-invariants have been useful in answering a range of questions in low-dimensional topology. A nice source of rational homology 3-spheres comes from considering double branched covers $\Sigma_2(K)$ of knots K in S^3. If $\Sigma_2(K)$ is an L-space, then the d-invariant of $\Sigma_2(K)$, at the unique spin-structure $\mathfrak{s}_0$, is well-understood: Lin-Ruberman-Saveliev in 2020 showed that it's a multiple of the signature of K.

When the branch set is a quasi-alternating link, the d-invariants of the double branched cover can be recovered from the signatures of the link in a similar way; this is due to Lisca-Owens in 2015. In this talk, we show that a similar phenomenon holds for branching over certain families of non-quasi-alternating links. This is work in progress with M. Marengon.

CLAUDIUS ZIBROWIUS, University of Regensburg
Khovanov homology and strong inversions

There is a one-to-one correspondence between strong inversions on knots in the three-sphere and a special class of four-ended tangles. I will discuss recent work with Artem Kotelskiy and Liam Watson in which we compute the reduced Khovanov homology of such tangles for all strong inversions on knots with up to 9 crossings [arXiv: 2104.13592].