Harmonic Analysis and Partial Differential Equations
Analyse harmonique et équations différentielles partielles
(Org: **Almaz Butaev** (Calgary) and/et **Galia Dafni** (Concordia))

RYAN ALVARADO, Amherst College
Optimal embeddings and extensions for Triebel-Lizorkin spaces in spaces of homogeneous type

Embedding and extension theorems for certain classes of function spaces in $\mathbb{R}^n$ (such as Sobolev spaces) have played a fundamental role in the area of partial differential equations. In this talk, we will discuss some recent work which builds upon such results and identifies necessary and sufficient conditions guaranteeing that certain Sobolev-type inequalities and extension results hold for the scale of Triebel-Lizorkin spaces ($M^s_{p,q}$ spaces) in the general context of spaces of homogeneous type. An interesting facet of this work is how the range of s (the smoothness parameter) for which these inequalities and extension results hold is intimately linked to the geometric makeup of the underlying space. This talk is based on joint work with Dachun Yang and Wen Yuan.

TATYANA BARRON, University of Western Ontario
Weighted Bergman spaces on the ball and submanifolds

I will talk about certain finite-dimensional subspaces of weighted Bergman spaces on the unit ball in $\mathbb{C}^n$. Informally speaking, to a submanifold M of the ball one can associate an element f of this function space, by integrating the Bergman kernel over M. I will talk about norm estimates on f, and how they reflect the geometry of M.

ZACHARY BRADSHAW, University of Arkansas
Non-decaying solutions to the critical surface quasi-geostrophic equations with symmetries

We discuss a theory of self-similar solutions to the critical surface quasi-geostrophic equations due to Dallas Albritton and Z.B. In particular, we examine a construction of self-similar solutions for arbitrarily large data in various regularity classes—including some large, unbounded, non-decaying functions—and demonstrate, in the small data regime, uniqueness and global asymptotic stability. These solutions are non-decaying at spatial infinity, which leads to ambiguity in the drift velocity. This ambiguity is corrected by imposing m-fold rotational symmetry. The self-similar solutions of interest lie just beyond the known well-posedness theory and are expected to shed light on potential non-uniqueness, due to the possibility of symmetry-breaking bifurcations.

SAGUN CHANILLO, Rutgers University
Local Version of Courant's Nodal Domain Theorem

Given a compact Riemannian manifold with no boundary (M^n, g) endowed with a smooth metric g, one of the important objects of study is the Laplace-Beltrami operator and its eigenfunctions. That is

$$-\Delta u_k = \lambda_k u_k.$$

The Courant nodal domain theorem asserts that the k-th eigenfunction has at most k nodal domains, where a nodal domain is a connected component of the set $\{x|\ u_k(x) \neq 0\}$. Harold Donnelly and C. Fefferman initiated the study of local versions of this result with a goal to show that nodal domains cannot be long and narrow. This was related to a conjecture of S.-T. Yau on the length of the nodal set. The nodal set is the set $\{x|\ u_k(x) = 0\}$. In this joint work with A. Logunov, E. Mallinikova and D. Mangoubi, we obtain an optimal bound for results of this type.

DAVID CRUZ-URIBE, The University of Alabama
Sharp constant estimates for matrix weighted inequalities

In this talk we will review some recent work on strong (p,p) and weak $(1,1)$ inequalities with matrix weights: e.g., inequalities of the form

$$\int_{\mathbb{R}^n} |W^{1/p}(x)T\mathbf{f}(x)|^p\,dx \leq C\int_{\mathbb{R}^n} |W^{1/p}(x)\mathbf{f}(x)|^p\,dx,$$

$$|\{x\in\mathbb{R}^n : |W(x)T(W^{-1}\mathbf{f})(x)| > t\}| \leq \frac{C}{t}\int_{\mathbb{R}^n} |\mathbf{f}(x)|\,dx.$$

W is an $n\times n$ self-adjoint, positive semi-definite matrix that satisfies the matrix A_p condition, and T is a Calderón-Zygmund operator. We will also mention results for the Golderg maximal operator and commutators. We will conclude with some open questions in both the scalar and matrix weighted cases.

This is joint work with Kabe Moen, Josh Isralowitz, Sandra Pott and Israel Rivera-Rios.

GUY C. DAVID, Ball State University
Quantitative decompositions of Lipschitz mappings

Given a Lipschitz map, it is often useful to chop the domain into pieces on which the map has simple behavior. For example, depending on the dimensions of source and target, one may ask for pieces on which the map behaves like a bi-Lipschitz embedding or like a linear projection. It is even more useful if this decomposition is quantitative, i.e., with bounds independent of the particular map or spaces involved. After surveying the question of bi-Lipschitz decomposition, we will discuss the more complicated case in which dimension decreases, e.g., for maps from $\mathbb{R}^3$ to $\mathbb{R}^2$. This is joint work with Raanan Schul, improving a previous result of Azzam-Schul.

RYAN GIBARA, Université Laval
Dyadic structure theorems for strong function spaces

The space $BMO(\mathbb{R}^n)$ can be shown to coincide with the intersection of N dyadic-type BMO spaces, where $N>1$. Moreover, it is known that the sharp (i.e. smallest possible) value for N is $n+1$. In joint work with José Conde-Alonso, we consider the case of strong $BMO(\mathbb{R}^n)$, where mean oscillation is bounded over all rectangles with sides parallel to the axes. We exploit the product structure inherent to rectangles and inherited by strong $BMO(\mathbb{R}^n)$ to show that an analogous result holds for this function space with $N=2$ regardless of the dimension. Other function spaces such as BLO and VMO are also considered.

PAUL HAGELSTEIN, Baylor University
On the finiteness of strong maximal functions associated to functions whose integrals are strongly differentiable

Besicovitch proved that if f is an integrable function on $\mathbb{R}^2$ whose associated strong maximal function $M_S f$ is finite a.e., then the integral of f is strongly differentiable. On the other hand, Papoulis proved the existence of a function in $L^1(\mathbb{R}^2)$ (taking on both positive and negative values) whose integral is strongly differentiable but whose associated strong maximal function is infinite on a set of positive measure. In this talk, we discuss a recent result of Hagelstein and Oniani that if $f\in L^1(\mathbb{R}^n)$ is a *nonnegative* function whose integral is strongly differentiable and moreover such that $f(1+\log^+ f)^{n-2}$ is integrable, then $M_S f$ is finite a.e. This result is sharp in that, if ϕ is a convex increasing function on $[0,\infty)$ such that $\phi(0)=0$ and with $\phi(u)=o(u(1+\log^+ u)^{n-2})$ $(u\to\infty)$, then there exists a nonnegative function f on $\mathbb{R}^n$ such that $\phi(f)$ is integrable on $\mathbb{R}^n$ and the integral of f is strongly differentiable, although $M_S f$ is infinite on a set of positive measure.

RITVA HURRI-SYRJÄNEN, University of Helsinki
On the John-Nirenberg Space

The talk will address 'from local to global' questions for functions in the John-Nirenberg space. When inequalities are known to be true locally, we will discuss corresponding global results for functions defined in bounded domains. My talk is based on joint work with Niko Marola and Antti V. Vähäkangas.

DAMIR KINZEBULATOV, Université Laval
Heat kernel bounds and stochastic equations with singular (form-bounded) drift

I will talk about recent results on sharp two-sided heat kernel bounds for divergence-form parabolic equations with drift having critical singularities, and related results on stochastic differential and stochastic transport equations. The talk is based on joint papers with K.R.Madou, Yu.A.Semenov and R.Song.

LUDA KOROBENKO, Reed College
Continuity of weak solutions via the trace method

In this talk I will discuss some new regularity results for weak solutions to infinitely degenerate elliptic equations on the plane. The main result is continuity of weak solutions for operators that have bounded measurable coefficients and are only comparable to the diagonal operator of the form $\partial_x^2 + f^2(x)\partial_y^2$, which can be seen as a generalization of Fediĭ's remarkable hypoellipticity theorem. To establish this result, we develop a trace method that first constructs a region in $\mathbb{R}^2$ on whose boundary a given subsolution u has a suitable trace, and then applies a maximum principle to derive local boundedness and continuity of weak solutions.

MARTA LEWICKA, University of Pittsburgh
On the Monge-Ampere system

The Monge-Ampere equation $\det \nabla^2 u = f$ posed on a $N = 2$ dimensional domain, has a natural weak formulation that appears as the constraint condition in the Γ-limit of the dimensionally reduced non-Euclidean elastic energies. This formulation reads: $curl^2(\nabla v \otimes \nabla v) = -2f$ and it allows, via the Nash-Kuiper scheme of convex integration, for constructing multiple solutions that are dense in $C^0(\omega)$, at the regularity $C^{1,\alpha}$ for any $\alpha < 1/7$.

Does a similar result hold in higher dimensions $N > 2$? Indeed it does, but one has to replace the Monge-Ampere equation by a *"Monge-Ampere system"*, altering $curl^2$ to the corresponding operator whose kernel consists of the symmetrised gradients of N-dimensional displacement fields. We will show how this Monge-Ampere system arises from the prescribed Riemannian curvature problem by matched asymptotic expansions, similarly to how the prescribed Gaussian curvature problem leads to the Monge-Ampere equation in 2d, and prove that its flexibility at $C^{1,\alpha}$ for any $\alpha < 1/(N^2 + N + 1)$.

CLAUDIO MACHADO VASCONCELOS, Universidade Federal de São Carlos
On the continuity of Calderón-Zygmund-type operators on Hardy spaces

In this talk, we will discuss some boundedness results for strongly singular Calderón-Zygmund operators on Hardy spaces $H^p(\mathbb{R}^n)$ and its local version $h^p(\mathbb{R}^n)$ for $0 < p \leq 1$. Operators of this type are generalizations of *weakly-strongly multipliers* and include appropriate classes of pseudodifferential operators in the Hörmander class. In particular, we assume some L^s-type integral estimates on their kernel and present some interesting molecular decomposition of $h^p(\mathbb{R}^n)$, in which a weaker cancellation condition is assumed.

This is joint work with Tiago Picon (University of São Paulo), Galia Dafni and Chun Ho Lau (Concordia University).

TOMAS MERCHÁN, University of Minnesota
Huovinen transform and rectifiability

A major theorem of Tolsa, building upon prior work of Mattila-Preiss, states that if $E \subset \mathbb{R}^d$ with $\mathcal{H}^s(E) < \infty$ ($s \in \mathbb{Z}$), and the s-Riesz transform associated to E exists in principal value, then the set E is s-rectifiable. It has been an open problem if the analogous theorem holds in the case of the Huovinen transform (which has kernel $K(z) = z^k/|z|^{k+1}$ in $\mathbb{C}$ for k odd) for sets of positive and finite length. In the talk we will discuss this problem.

DORINA MITREA, Baylor University
A Sharp Divergence Theorem

In this talk I will discuss a version of the Divergence Theorem for vector fields which may lack any type of continuity and for which the boundary trace is taken in a strong, nontangential pointwise sense. These features of our brand of Divergence Theorem make it an effective tool in dealing with problems arising in various areas of mathematics, including Harmonic Analysis, Complex Analysis, Potential Analysis, and Partial Differential Equations. A few such applications will be presented.

MARIUS MITREA, Baylor University
Singular Integrals, Geometry of Sets, and Boundary Problems

Presently, it is well understood what geometric features are necessary and sufficient to guarantee the boundedness of convolution-type singular integral operators (SIO's) on Lebesgue spaces. This being said, dealing with other function spaces where membership entails more than a mere size condition (like Sobolev spaces, Hardy spaces, or the John-Nirenberg space BMO) requires new techniques. In this talk I will explore recent progress in this regard, and follow up the implications of such advances into the realm of boundary value problems.

VIRGINIA NAIBO, Kansas State University
Pseudo-multipliers on Hermite Besov and Hermite Triebel-Lizorkin spaces

We will present boundedness properties of pseudo-multipliers with symbols of Hörmander-type in function spaces associated to the Hermite operator. The main tools in the proofs involve new molecular decompositions and molecular synthesis estimates for Hermite Besov and Hermite Triebel-Lizorkin spaces, which allow to obtain boundedness results on spaces for which the smoothness allowed includes non-positive values. In particular, we obtain continuity results for pseudo-multipliers on Lebesgue and Hermite local Hardy spaces. This is based on joint work with Fu Ken Ly (The University fo Sydney).

SCOTT RODNEY, Cape Breton University
Iterations in PDEs

In this talk I will discuss some recent progress on joint work with D. Cruz-Uribe (University of Alabama) and S.F. MacDonald (CBU) concerning the boundedness of weak solutions to equations of the form

$$-\mathrm{Div}\,(Q(x)\nabla u(x)) = f(x)$$

in a bounded domain Ω of $\mathbb{R}^n$ with $n \geq 4$ and where $Q(x)$ is a symmetric non-negative definite matrix valued function on Ω. Using a De Giorgi iterative process we produce boundedness results for weak solutions u when the data function f belongs to an Orlicz class $L^\Psi(\Omega)$ where Ψ is a particular type of Young function satisfying $\Psi(t) > t^{n/2}$.

NAGES SHANMUGALINGAM, University of Cincinnati
Using hyperbolic fillings to connect Besov spaces of functions on doublling metric space to Sobolev functions on uniform domains

In this talk we will describe a way of identifying Besov spaces of functions on a compact doubling metric measure space as traces of Sobolev spaces on uniform domains. Functions in Besov spaces have non-local energy and so it is advantageous from the point of view of regularity theory to associate them with more local energy spaces such as Sobolev spaces. This talk is based on joint work with Anders Bjorn and Jana Bjorn.

CODY STOCKDALE, Clemson University
Weighted theory of compact operators

The boundedness properties of singular integral operators are of central importance in analysis. Within the last decade, optimal bounds for general Calderón-Zygmund operators acting on weighted Lebesgue spaces in terms of Muckenhoupt weight characteristics have been obtained. In addition to this theory concerning boundedness, a theory for compactness of Calderón-Zygmund operators has recently been established. The first goal of this talk is to present the extension of compact Calderón-Zygmund theory to weighted spaces using sparse domination techniques. A similar line of research concerns the weighted boundedness of the Bergman projection in terms of Bekollé-Bonami weights, and compactness in this setting can be understood within the study of Toeplitz operators. We also discuss the weighted theory of Toeplitz operators on the Bergman space.

ALEX STOKOLOS, Georgia Southern University
"An extremal problem for polynomials"

In 1987 M.Brandt solved the extremal problem

$$\sup_{a_2,\ldots,a_N} \left(\inf_{z\in\mathbb{D}} \{\Re\left(F\left(z\right)\right) : \Im\left(F\left(z\right)\right) = 0\} \right)$$

for the univalent in $\mathbb{D}$ polynomials $F(z) = \sum_{j=1}^{N} a_j z^j$ with real coefficients and normalization $a_1 = 1$. He proved that the solution is $-\frac{1}{4}\sec^2\frac{\pi}{N+2}$, and found the extremal polynomial. We prove that the above problem stated for general (not necessary univalent) polynomials has the same solution and the same extremizer. Moreover, we prove the uniqueness of the extremizer and obtain the estimate on the Koebe radius for polynomials in various settings. This is a joint work with Dmitriy Dmitrishin and Andrey Smorodin.

KRYSTAL TAYLOR, The Ohio State Math Department
Quantifications of the Besicovitch Projection theorem in a nonlinear setting

There are many classical results relating the geometry, dimension, and measure of a set to the structure of its orthogonal projections. It turns out that many nonlinear projection-type operators also have special geometry that allows us to build similar relationships between a set and its "projections," just as in the linear setting. We will discuss a series of recent results from both geometric and probabilistic vantage points. In particular, we will see that the multi-scale analysis techniques of Tao, as well as the energy techniques of Mattila, can be strengthened and generalized to projection-type operators satisfying a transversality condition. As an application, we find upper and lower bounds for the rate of decay of the Favard curve length of the four-corner Cantor set.

IGNACIO URIARTE-TUERO, University of Toronto
Two weight norm inequalities for singular and fractional integral operators in $\mathbb{R}^n$

I will report on recent progress on the two weight problem for singular and fractional integral operators in $\mathbb{R}^n$, in particular a two weight local Tb theorem in higher dimensions.

Joint work with Christos Grigoriadis, Michalis Paparizos, Eric Sawyer, Chun-Yen Shen.

JEAN VAN SCHAFTINGEN, UCLouvain

Marcinkiewicz meets Gagliardo and Sobolev: weak-type formulas for norms of the gradient

I will present new results characterising Sobolev norms of functions with Marcinkiewicz weak-type estimates for the integrand of the Gagliardo semi-norm and their application to detection of constant functions and repairing the fractional Gagliardo–Nirenberg interpolation at endpoints where it fails.

This is a joint work with Haim Brezis (Rutgers, Technion Haifa and Sorbonne) and Po Lam Yung (Chinese University of Hong Kong and Australian National University).

J. MICHAEL WILSON, University of Vermont

Perturbation of dyadic averages

If $f : \mathbf{R}^d \to \mathbf{C}$ is locally integrable and $E \subset \mathbf{R}^d$ is bounded and measurable, with positive Lebesgue measure $|E|$, then f_E means f's average over E: $f_E := \frac{1}{|E|}\int_E f\,dt$. $\mathcal{D}$ denotes the family of dyadic cubes in $\mathbf{R}^d$. By the Lebesgue Differentiation Theorem, for a.e. $x \in \mathbf{R}^d$, $f_Q \to f(x)$ as $|Q| \to 0$, for $Q \in \mathcal{D}$ such that $x \in Q$. Suppose that, for some fixed $0 < \eta \ll 1$, and for every $Q \in \mathcal{D}$, we have an $n \times n$ real matrix $A^{(Q)}$ and a vector $y^{(Q)} \in \mathbf{R}^d$ such that: a) $\|I_d - A^{(Q)}\|_\infty < \eta$, where I_d is the identity matrix and $\|\cdot\|_\infty$ is the standard matrix norm; b) $|y^{(Q)}| \leq \eta$. For each $Q \in \mathcal{D}$ define

$$\begin{aligned} F^{(Q)}(x) &:= \chi_Q\left(A^{(Q)}(x - x_Q + \ell(Q)y^{(Q)}) + x_Q\right) \\ &=: \chi_{Q^*}(x), \end{aligned}$$

where x_Q is Q's center. We think of Q^* as a perturbation of Q resulting from a close-to-the-identity affine transformation "centered" on x_Q. The averages f_{Q^*} converge to a.e. x as $|Q| \to 0$ for $x \in Q \in \mathcal{D}$.

Elementary estimates with the Hardy-Littlewood maximal function show that, for all $s > 2$, there are constants $c(d) > 0$ and $C(d, s)$ so that if $\eta < c(d)$ then, for all $f \in L^2(\mathbf{R}^d)$,

$$\left\| \sup_{x \in Q \in \mathcal{D}} |f_Q - f_{Q^*}| \right\|_2 \leq C(d,s)\eta^{1/s}\|f\|_2.$$

We improve this to get: *There are constants $c(d) > 0$ and $C(d)$ so that if $\eta < c(d)$ then, for all $f \in L^2(\mathbf{R}^d)$,*

$$\left\| \left(\sum_{x \in Q \in \mathcal{D}} |f_Q - f_{Q^*}|^2 \right)^{1/2} \right\|_2 \leq C(d)\eta^{1/2}\|f\|_2.$$