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On the Monge-Ampere system

The Monge-Ampere equation $\det \nabla^2 u = f$ posed on a $N = 2$ dimensional domain, has a natural weak formulation that appears as the constraint condition in the Γ-limit of the dimensionally reduced non-Euclidean elastic energies. This formulation reads: $curl^2(\nabla v \otimes \nabla v) = -2f$ and it allows, via the Nash-Kuiper scheme of convex integration, for constructing multiple solutions that are dense in $C^0(\omega)$, at the regularity $C^{1,\alpha}$ for any $\alpha < 1/7$.

Does a similar result hold in higher dimensions $N > 2$? Indeed it does, but one has to replace the Monge-Ampere equation by a *"Monge-Ampere system"*, altering $curl^2$ to the corresponding operator whose kernel consists of the symmetrised gradients of N-dimensional displacement fields. We will show how this Monge-Ampere system arises from the prescribed Riemannian curvature problem by matched asymptotic expansions, similarly to how the prescribed Gaussian curvature problem leads to the Monge-Ampere equation in 2d, and prove that its flexibility at $C^{1,\alpha}$ for any $\alpha < 1/(N^2 + N + 1)$.