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Heffter arrays and biembeddings of cycle systems

In the last 20 years biembedding pairs of designs and cycle systems onto surfaces has been a muchresearched topic (see the 2007 survey "Designs and Topology" by Grannell and Griggs). In particular, in a posthumous work (2015), Archdeacon showed that biembeddings of cycle systems may be obtained via Heffter arrays. Formally, a Heffter array $H(m, n; s, t)$ is an $m \times n$ array of integers such that: (a) each row contains s filled cells and each column contains t filled cells; (b) the elements in every row and column sum to 0 in $\mathbb{Z}_{2ms+1}$; and (c) for each integer $1 \leq x \leq ms$, either x or $-x$ appears in the array. If we can order the entries of each row and column satisyfing two properties (compatible and simple), a Heffter array yields an embedding of two cycle decompositions of the complete graph K_{2ms+1} onto an orientable surface. Such an embedding is face 2-colourable, where the faces of one colour give a decomposition into s-cycles and the faces of the other colour gives a decomposition into t-cycles. Thus as a corollary the two graph decompositions are orthogonal; that is, any two cycles share at most one edge. Moreover, the action of addition in $\mathbb{Z}_{2ms+1}$ gives an automorphism of the embedding. We give more detail about the above and present a new result: the existence of Heffter arrays $H(n, n; s, s)$ with compatible and simple orderings whenever $s \equiv 3 \pmod 4$ and $n \equiv 1 \pmod 4$.