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Towards higher dimensional Gromov compactness in G_2 and Spin(7) manifolds

Let (M, ω) be a compact symplectic manifold with a compatible almost complex structure J. We can study the space of J-holomorphic maps $u\colon \Sigma \to (M, J)$ from a compact Riemann surface into M. By "compactifying" the space of such maps, one can obtain powerful global symplectic invariants of M. This requires understanding the ways in which sequences of such maps can develop singularities. Crucial ingredients are conformal invariance and an energy identity, which lead to to a plethora of analytic consequences, including: (i) a mean value inequality, (ii) interior regularity, (iii) a removable singularity theorem, (iv) an energy gap, and (v) compactness modulo bubbling.

Riemannian manifolds with closed G_2 or $\mathrm{Spin}(7)$ structures share many similar properties to such almost Kahler manifolds. In particular, they admit analogues of J-holomorphic curves, called associative and Cayley submanifolds, respectively, which are calibrated and hence homologically volume-minimizing. A programme initiated by Donaldson-Thomas-Segal aims to construct similar such "counting invariants" in these cases. In 2011, an overlooked preprint of Aaron Smith demonstrated that such submanifolds can be exhibited as images of a class of maps $u\colon \Sigma \to M$ satisfying a conformally invariant first order nonlinear PDE analogous to the Cauchy-Riemann equation, which admits an energy identity involving the integral of higher powers of the pointwise norm $|du|$. I will discuss joint work (to appear in Asian J. Math.) with Da Rong Cheng (Waterloo) and Jesse Madnick (NCTS/NTU) in which we establish the analogous analytic results of (i)-(v) in this setting. arXiv:1909.03512