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Impartial games with entailing moves

Combinatorial Game Theory has also been called 'additive game theory', whenever the analysis involves sums of independent game components. Such *disjunctive sums* invoke comparison between games, which allows abstract values to be assigned to them. However, there are rulesets with *entailing moves* that break the alternating play axiom and/or restrict the other player's options within the disjunctive sum components. These situations are exemplified in the literature by a ruleset such as Nimstring, a normal play variation of the classical children's game Dots and Boxes, and Top Entails, an elegant ruleset introduced in the classical work *Winning Ways*, by Berlekamp Conway and Guy. Such rulesets fall outside the scope of the established normal play theory. Here, we axiomatize normal play via two new terminating games, Inf (Left wins) and -Inf (Right wins), and a more general theory is achieved. We define *affine impartial*, which extends classical impartial games, and we analyze their algebra by extending the established Sprague-Grundy theory, with an accompanying minimum excluded rule. Solutions of Nimstring and Top Entails are given to illustrate the theory.