TAKASHI OWADA, Purdue University

Convergence of persistence diagram in the subcritical regime

The objective of this work is to examine the asymptotic behavior of persistence diagrams associated with Čech filtration. A persistence diagram is a graphical descriptor of a topological and algebraic structure of geometric objects. We consider Čech filtration over a scaled random sample $r_n^{-1}\mathcal{X}_n = \{r_n^{-1}X_1, \dots, r_n^{-1}X_n\}$, such that $r_n \to 0$ as $n \to \infty$. We treat persistence diagrams as a point process and establish their limit theorems in the subcritical regime: $nr_n^d \to 0$, $n \to \infty$. In this setting, we show that the asymptotics of the kth persistence diagram depends on the limit value of the sequence $n^{k+2}r_n^{d(k+1)}$. If $n^{k+2}r_n^{d(k+1)} \to \infty$, the scaled persistence diagram converges to a deterministic Radon measure almost surely in the vague metric. If r_n decays faster so that $n^{k+2}r_n^{d(k+1)} \to c \in (0, \infty)$, the persistence diagram weakly converges to a limiting point process without normalization. Finally, if $n^{k+2}r_n^{d(k+1)} \to 0$, the sequence of probability distributions of a persistence diagram should be normalized, and the resulting convergence will be treated in terms of the $\mathcal{M}_0$-topology.