SEDI BARTZ, University of Massachusetts Lowell
Monotone operators and convex analysis in multi-marginal settings

Motivated by multi-marginal optimal transport, we discuss aspects of its underlying monotone and convex analytic structure. We review recent extensions of classical maximal monotonicity and convex analysis into multi-marginal settings. We point out several open questions as well as partial resolutions and examples. In particular, we focus on an extension of Minty's characterization of maximal monotonicity and a generalization of the maximal monotonicity of the convex subdifferential.

HEINZ BAUSCHKE, UBC Okanagan
Compositions of projection mappings: fixed point sets and difference vectors

Projection operators and associated projection algorithms are fundamental building blocks in fixed point theory and optimization.

In this talk, I will survey recent results on the displacement mapping of the right-shift operator and sketch a new application deepening our understanding of the geometry of the fixed point set of the composition of projection operators in Hilbert space.

Based on joint works with Salha Alwadani, Julian Revalski, and Shawn Wang.

WALAA MOURSI, University of Waterloo
Further notions of monotonicity and corresponding properties of resolvents and reflected resolvents

The correspondence between the monotonicity of a (possibly) set-valued operator and the firm nonexpansiveness of its resolvent is a key ingredient in the convergence analysis of many optimization algorithms. Firmly nonexpansive operators form a proper subclass of the more general - but still pleasant from an algorithmic perspective - class of averaged operators. In this talk, we introduce the notion of conically nonexpansive operators which generalize nonexpansive mappings. We characterize averaged operators as being resolvents of comonotone operators under appropriate scaling. As a consequence, we characterize the proximal point mappings associated with hypoconvex functions as cocoercive operators, or equivalently; as displacement mappings of conically nonexpansive operators. Several examples illustrate our analysis and demonstrate the tightness of our results.

HUI OUYANG, University of British Columbia, Okanagan
Bregman Circumcenters

In this talk, we first characterize backward and forward Bregman projections onto affine subspaces. Then we introduce backward and forward Bregman (pseudo-)circumcenter operators associated with finite sets. We also demonstrate the existence of backward and forward Bregman (pseudo-)circumcenters of finite sets and show explicit formulae for the unique backward and forward Bregman pseudo-circumcenters of finite sets. Moreover, we state some dual expressions of the backward and forward Bregman (pseudo-)circumcenters of finite sets. Various examples are presented to illustrate the backward and forward Bregman (pseudo-)circumcenters of finite sets. In particular, one example illuminates the existence of the backward Bregman pseudo-circumcenter, while the traditional circumcenter under the Euclidean distance does not exists.

HUNG PHAN, University of Massachusetts Lowell
Adaptive Splitting Algorithms

A general optimization problem can often be reduced to finding a zero of a sum of multiple (maximally) monotone operators, which creates challenging computational tasks as a whole. It motivates the development of splitting algorithms in order to simplify the computations by dealing with each operator separately. Some of the most successful splitting algorithms in applications are the forward-backward algorithm, the Douglas-Rachford algorithm, and the alternating directions method of multipliers (ADMM). In this talk, we discuss some adaptive generalizations of such splitting algorithms. The main idea is to adapt the algorithm parameters to the properties of each operators so that the generated sequences converge to a fixed point.

HRISTO SENDOV, The University of Western Ontario
A unified approach to operator monotone functions

The notion of operator monotonicity dates back to a work by Löwner in 1934. A map $F : S^n \to S^m$ is called *operator monotone*, if $A \succeq B$ implies $F(A) \succeq F(B)$. (Here, S^n is the space of symmetric matrices with the semidefinite partial order $\succeq$.) Often, the function F is defined in terms of an underlying simpler function f. Of main interest is to find the properties of f that characterize operator monotonicity of F. In that case, it is said that f is also operator monotone. Classical examples are the Löwner operators and the spectral (scalar-valued isotropic) functions. Operator monotonicity for these two classes of functions is characterized in seemingly very different ways.

This talk extends the notion of operator monotonicity to symmetric functions f on k arguments. The latter is used to define *(generated) k-isotropic maps* $F : S^n \to S^{\binom{n}{k}}$ for any $n \geq k$. Necessary and sufficient conditions are given for f to generate an operator monotone k-isotropic map F. When $k = 1$, the k-isotropic map becomes a Löwner operator and when $k = n$ it becomes a spectral function. This allows us to reconcile and explain the differences between the conditions for monotonicity for the Löwner operators and the spectral functions.

SHAMBHAVI SINGH, UBC Okanagan
Finding Best Approximation Pairs for Two Intersections of Closed Convex Sets

The problem of finding a best approximation pair of two sets, which in turn generalizes the well known convex feasibility problem, has a long history that dates back to work by Cheney and Goldstein in 1959.

In 2018, Aharoni, Censor, and Jiang revisited this problem and proposed an algorithm that can be used when the two sets are finite intersections of halfspaces. Motivated by their work, we present alternative algorithms that utilize projection and proximity operators. Numerical experiments indicate that these methods are competitive and sometimes superior to the one proposed by Aharoni et al.

XIANFU WANG, University of British Columbia
Attouch-Thera Duality, Generalized Cycles and Gap Vectors

Using the Attouch-Thera duality, we study the cycles, gap vectors and fixed point sets of compositions of proximal mappings. A primal-dual framework provides an exact relationship between the cycles and gap vectors. We also introduce the generalized cycle and gap vectors to tackle the case when the classical ones do not exist. Joint work with Bauschke and Alwadani.

ZIYUAN WANG, UBC Okanagan
Calculus rules of the generalized Kurdyka-Łojasiewicz property

In this work, we propose several calculus rules for the generalized Kurdyka-Łojasiewicz (KL) property, some of which generalize Li and Pong's results. We have shown in a parallel paper that the optimal desingularizing function has various forms and may be nondifferentiable. Our calculus rules do not assume desingularizing functions to have any specific form or differentiable, while the known results do. Several examples are also given to show that our calculus rules are applicable to a broader class of functions than the known ones.

JANE YE, University of Victoria
Difference of convex algorithms for bilevel programs with applications in hyperparameter selection

In this paper, we present difference of convex algorithms for solving bilevel programs in which the upper level objective functions are difference of convex functions, and the lower level programs are fully convex. This nontrivial class of bilevel programs provides a powerful modelling framework for dealing with applications arising from hyperparameter selection in machine learning. Thanks to the full convexity of the lower level program, the value function of the lower level program turns out to be convex and hence the bilevel program can be reformulated as a difference of convex bilevel program. We propose two algorithms for solving the reformulated difference of convex program and show their convergence under very mild assumptions. Finally we conduct numerical experiments to a bilevel model of support vector machine classification.

YAO-LIANG YU, University of Waterloo
An Operator Splitting View of Federated Learning

Federated learning (FL) has recently emerged as a massively distributed framework that enables training a shared or personalized model without infringing user privacy. In this work, we show that many of the existing FL algorithms can be understood from an operator splitting point of view. This unification allows us to compare different algorithms with ease, to refine previous convergence results and to uncover new algorithmic variants. In particular, our analysis reveals the vital role played by the step size in FL algorithms. The unification also leads to a streamlined and economic way to accelerate FL algorithms, without incurring any communication overhead. We perform numerical experiments on both convex and nonconvex models to validate our findings.

JIM ZHU, Western Michigan University
Bank Balance Sheet Risk Allocation with Linear Programming

We discuss a bank balance sheet risk allocation problem arising in practical banking setting. This problem involves two different kinds of risks and, therefore, does not fit into the traditional portfolio model. We solve the problem with linear programming using duality. Both primal and dual solutions have intuitive financial explanations and provides insight for practical balance sheet management strategies. This is a joint work with Dr. Judice from Montepio Bank, Portugal.