Stochastic partial differential equations
Équations aux dérivées partielles stochastiques

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MIKE KOURITZIN, University of Alberta

Local interactions in stochastic differential equations

This talk with be based on joint work with Tom Kurtz and Jie Xiong.

An infinite system of real-valued stochastic differential equations for particle locations is considered. The particles exhibit local interactions through drift coefficients that depend upon other particles within a fixed distance. Strong existence and uniqueness is proved for this particle system with potentially discontinuous, local interactions. Current related work in stochastic partial differential equations will be discussed if time permits.

CARL MUELLER, University of Rochester

A Small Ball Problem for the Random String

This is joint work with Siva Athreya and Mathew Joseph.

Small ball problems for stochastic processes have a long history. One seeks to estimate the probability that a process stays in a small ball for a long time. Such estimates help us study the Hausdorff measure of the range of the process, among other things. Most such results involve Markov processes taking values in finite dimensional spaces, or Gaussian random fields. We establish a small ball estimate for vector-valued solutions of the stochastic heat equation with multiplicative white noise, which falls outside of the class of processes mentioned above. At one point we need to use the best constant in the Burkholder-Davis-Gundy inequality for large values of p.

DAVID NUALART,

Convergence of densities for the stochastic heat equation

Consider the one-dimensional stochastic heat equation driven by a space-time white noise with constant initial condition. The purpose of this talk is to present a recent result on the uniform convergence of the density of the normalized spatial averages of the solution on an interval $[-R,R]$, as R tends to infinity, to the density of the standard normal distribution, assuming some non-degeneracy and regularity conditions on the nonlinear coefficient σ. The proof is based on the combination of the techniques of Malliavin calculus with Stein's method for normal approximations.

MARKUS RIEDLE, King's College London

Stochastic evolution equations driven by cylindrical stable noise

In this talk we present an existence result for the mild solution of a stochastic evolution equation driven by a symmetric α-stable cylindrical Lévy process defined on a Hilbert space for $\alpha \in (1,2)$. In contrast to other literature, our work is based on the so-called semigroup approach to SPDEs. Similar to the fact that there are no standard Gaussian distribution in an infinite dimensional Hilbert space, the symmetric α-stable noise only exists in a generalised sense. As a consequence, to derive the existence result, we need to employ some non-standard methods, which we will present and discuss in this talk.

Joint work with Tomasz Kosmala.

MICKEY SALINS, Boston University

Global solutions for the stochastic reaction-diffusion equation with polynomially dissipative forcing

We identify a condition that implies that solutions to the stochastic reaction-diffusion equation $\frac{\partial u}{\partial t} = \mathcal{A}u + f(u) + \sigma(u)\dot{W}$ on a bounded spatial domain never blow up. We consider the case where f features polynomial dissipativity of the form $f(u)\text{sign}(u) \leq \breve{}K_1|u|^\beta$ for $\beta > 1$ and $|u|$ large. This kind of strong dissipation prevents solutions from blowing up even when the multiplicative noise coefficient grows polynomially like $|\sigma(u)| \leq K_2(1+|u|^\gamma)$ as long as $\gamma < 1 + \frac{(1-\eta)(\beta-1)}{2}$. The constant $\eta \in (0,1)$ is a scalar that describes a relationship between the eigenvalues of the of linear operator and the eigenvalues of the Gaussian noise.

JIAN SONG, Shandong University
Scaling limit of a directed polymer among a Poisson field of independent walks

This talk is based on a joint work with Hao Shen, Rongfeng Sun and Lihu Xu.

We consider a directed polymer model in dimension $1+1$, where the disorder is given by the occupation field of a Poisson system of independent random walks on $\mathbb{Z}$. In a suitable continuum and weak disorder limit, we show that the family of quenched partition functions of the directed polymer converges to the Stratonovich solution of a multiplicative stochastic heat equation (SHE) with a Gaussian noise, whose space-time covariance is given by the heat kernel.

In contrast to the case with space-time white noise where the solution of the SHE admits a Wiener-Itô chaos expansion, we establish an L^1-convergent chaos expansions of iterated integrals generated by Picard iterations. Using this expansion and its discrete counterpart for the polymer partition functions, the convergence of the terms in the expansion is proved via functional analytic arguments and heat kernel estimates. The Poisson random walk system is amenable to careful moment analysis, which is an important input to our arguments.

XIAOMING SONG, Drexel University
Spatial averages for the Parabolic Anderson model driven by rough noise

In this paper, we study spatial averages for the parabolic Anderson model in the Skorohod sense driven by rough Gaussian noise, which is colored in space and time. We include the case of a fractional noise with Hurst parameters H_0 in time and H_1 in space, satisfying $H_0 \in (1/2, 1)$, $H_1 \in (0, 1/2)$ and $H_0 + H_1 > 3/4$. Our main result is a functional central limit theorem for the spatial averages. As an important ingredient of our analysis, we present a Feynman-Kac formula that is new for these values of the Hurst parameters.

WEI SUN, Concordia University
Periodic solutions of hybrid jump diffusion processes

We investigate periodic solutions of regime-switching jump diffusions. Uniqueness of periodic solutions to the corresponding SDEs or SPDEs is obtained by the strong Feller property and irreducibility of the associated time-inhomogeneous semigroups. Concrete examples are presented to illustrate the results. This talk is based on joint work with Xiao-Xia Guo and Chun Ho Lau.

DONGSHENG WU, University of Alabama in Huntsville
On Intersections of Independent Space-Time Anisotropic Gaussian Fields

Let $X^H = \{X^H(s), s \in \mathbb{R}^{N_1}\}$ and $X^K = \{X^K(t), t \in \mathbb{R}^{N_2}\}$ be two independent centered space-time anisotropic Gaussian random fields taking values in $\mathbb{R}^d$. In this talk, we study the existence of intersections of X^H and X^K. Furthermore, we determine the Hausdorff dimensions of the set of intersection times and the set of intersection points of the random fields, respectively.

This talk is based on a joint work with Zhenlong Chen and Jun Wang.

YIMIN XIAO, Michigan State University
Regularity Properties and Propagation of Singularities of the Stochastic Wave Equation

This talk is concerned with the regularity properties of the solution of the stochastic wave equation with additive Gaussian noise which is white in time and homogeneous in space. We show that the solution $\{u(t,x), t \geq 0, x \in \mathbb{R}\}$, as a Gaussian random field, has the property of sectorial local nondeterminism. Based on this property, we establish the exact uniform modulus of continuity for the solution.

We also study the problem of "propagation of singularities" for the solution $\{u(t,x), t \geq 0, x \in \mathbb{R}\}$. Our approach is based on a simultaneous law of the iterated logarithm and general methods for Gaussian processes.

This talk is based on joint papers with Cheuk-Yin Lee.

JIANLIANG ZHAI, University of Science and Technology of China

Large and moderate deviation principles for McKean-Vlasov SDEs with jumps

We consider McKean-Vlasov stochastic differential equations (MVSDEs) driven by Lévy noise. By identifying the right equations satisfied by the solutions of the MVSDEs with shifted driving Lévy noise, we build up a framework to fully apply the weak convergence method to establish large and moderate deviation principles for MVSDEs. In the case of ordinary SDEs, the rate function is calculated by using the solutions of the corresponding skeleton equations simply replacing the noise by the elements of the Cameron-Martin space. It turns out that the correct rate function for MVSDEs is defined through the solutions of skeleton equations replacing the noise by smooth functions and replacing the distributions involved in the equation by the distribution of the solution of the corresponding deterministic equation (without the noise). This is somehow surprising. With this approach, we obtain large and moderate deviation principles for much wider classes of MVSDEs in comparison with the existing literature (see AAP 29(2019),1487-1540). This talk is based on joint work with Wei Liu, Yulin Song, and Tusheng Zhang.

XIAOWEN ZHOU, Concordia University

Boundary behaviors for continuous-state nonlinear branching processes

We consider a class of continuous-state branching processes with nonadditive branching mechanism. Such a process arises as nonnegative solution to a generalized version of the stochastic differential equation (driven by both a Brownian motion and a spectrally positive Poisson random measure) for the classical continuous-state branching process. We present quite sharp conditions on parameters of the SDE under which extinction, explosion or coming down from infinity occurs, respectively, to these processes.

The talk is based on joint work with Pei-Sen Li and Xu Yang.