PETER KRISTEL, University of Manitoba

The smooth spinor bundle on loop space

Given a smooth manifold, M, there is a hierarchy of interesting extra structures that M may or may not admit: $\mathrm{metric} \leftarrow \mathrm{orientation} \leftarrow \mathrm{spin\ structure} \leftarrow \mathrm{string\ structure} \leftarrow \dots$, these structures correspond to reductions of the structure group of TM along the Whitehead tower of the orthogonal group $\mathrm{GL}(d) \cong \mathrm{O}(d) \leftarrow \mathrm{SO}(d) \leftarrow \mathrm{Spin}(d) \leftarrow \mathrm{String}(d) \leftarrow \dots$. Manifolds which admit a spin structure have extremely rich geometry, and are still being studied intensively. Manifolds with a string structure, on the other hand, are not nearly as well understood. One of the main difficulties is that $\mathrm{String}(d)$ is not a Lie group. In the eighties, Killingback argued that a string structure on M induces a spin structure on the smooth loop space $LM = C^\infty(S^1, M)$. Seemingly, this exchanges one difficulty for another, because LM is infinite dimensional, and classical spin geometry does not apply. In this talk I will explain how to adapt one of the fundamental notions of spin geometry, namely the spinor bundle, to this infinite dimensional case.