TOM HAINES, University of Maryland
Geometry of affine Schubert varieties and applications

Classical Schubert varieties are orbit-closures of a Borel subgroup acting on a partial flag variety attached to a connected reductive group. They play a central role in representation theory and combinatorics. Their geometric properties – whether they are normal, Cohen-Macaulay, or Frobenius-split; when they are singular, and what kind of singularities arise, etc – have been intensively studied and are now well understood. Affine Schubert varieties are similar objects but attached to a loop group rather than a group. They play a role in representation theory, mathematical physics, and in geometric approaches to automorphic forms. In the last 20 years they have been studied in large part because of their connection to certain Shimura varieties through the theory of Rapoport-Zink local models. But some key geometric properties – including normality – remain somewhat mysterious to this day, at least in some positive characteristic settings. This talk will survey some recent advances in the study of affine Schubert varieties, especially the surprising fact that almost all affine Schubert varieties in "bad" positive characteristic are not normal. We will connect this to the Langlands program by explaining how these results are used to understand the geometry of certain Shimura varieties.