Optimal transport and applications
Transport optimale et applications
(Org: **Young-Heon Kim** (UBC) and/et **Brendan Pass** (Alberta))

ALMUT BURCHARD, University of Toronto
How to differentiate functionals involving higher order derivatives along geodesics

I will describe work with Benjamin Schachter on differentiating functionals along Wasserstein geodesics, using an Eulerian point of view. The cost functions c(x,y) we consider are defined by minimizing the integral of a suitable Lagrangian among paths from x to y. We develop a formal procedure for computing derivatives of arbitrary order, and then appeal to the theory of transport equations (first-order linear PDE) to prove that the functionals vary smoothly along the geodesics, even when the density itself is not smooth. (Based on Ben's 2017 Ph.D. thesis)

SAMER DWEIK, University of British Columbia
Least gradient problem via optimal transport

In this talk, we consider the least gradient problem with Dirichlet condition imposed on (a part of) the boundary. In 2D, we show that this problem is equivalent to an optimal transport problem. Thanks to this equivalence, we show existence and uniqueness of a solution to the least gradient problem and then, we prove some regularity on this solution by studying the summability of the transport density in the corresponding equivalent optimal transport formulation.

CHRISTIAN KETTERER, University of Toronto
Glued spaces and lower curvature bounds

First I will survey some classical theorems about glued spaces and lower curvature bounds for Riemannian manifolds. Then I will present a recent result together with Vitali Kapovitch and Karl-Theodor Sturm showing that in the class of Alexandrov spaces equipped with a semi-concave weight the Riemannian curvature-dimension condition (RCD) is preserved under gluing constructions with optimal lower curvature bounds. The RCD condition, a synthetic notion of Ricci curvature bounded from below, is introduced by means of optimal transport.

MAXIME JACKY P. LABORDE, Université de Paris
An augmented Lagrangian method for transportation distance with bulk/interface interactions

Recently, Monsaingeon introduced a new optimal transport problem on a closed bounded domain defined via a dynamical Benamou-Brenier formulation. The model handles differently the motion in the interior and on the boundary, and penalizes the transfer of mass between the two. Taking advantage of the dynamical formulation, in this talk we will present a numerical method to compute this problem using an augmented Lagrangian method. This algorithm extends the ALG2 method introduced by Benamou-Brenier to solve the classical optimal transport problem. This is a joint work with Thomas Gallouët and Léonard Monsaingeon.

HUGO LAVENANT, Bocconi University
The Branching Schrödinger Problem

It is now well understood that regularized (a.k.a. entropic) optimal transport is linked to entropy minimization with respect to the law of the Brownian motion: this is the Schrödinger problem. I will present an ongoing work with Aymeric Baradat (CNRS, Lyon) where we explain how models of *regularized* unbalanced optimal transport are linked to entropy minimization with respect to the law of the *branching* Brownian motion.

NAM LE, Indiana University
Approximating minimizers of the Rochet-Chone functional with non-quadratic costs by solutions of singular Abreu equations

The Rochet-Chone model for the monopolist problem in product line design is a variational problem with a convexity constraint. This constraint renders serious challenges in numerically computing its minimizers, and calls for robust approximation schemes. In this talk, we show that, for a full range of q, minimizers of the Rochet-Chone functional with a convexity constraint in two dimensions can be approximated in the uniform norm by solutions of singular, fourth order Abreu equations that arise in extremal metrics in complex geometry.

ROBERT MCCANN, University of Toronto
Maximizing the sum of angles between pairs of lines in Euclidean space

Choose N unoriented lines through the origin of $\mathbf{R}^{d+1}$. Suppose each pair of lines repel each other with a force whose strength is independent of the (acute) angle between them, so that they prefer to be orthogonal to each other. However, unless $N \leq d+1$, it is impossible for all pairs of lines to be orthogonal. What then are their stable configurations? An unsolved conjecture of Fejes Toth (1959) asserts that the lines should be equidistributed as evenly as possible over an orthonormal basis in $\mathbf{R}^{d+1}$. By modifying the force to make it increase as a power of the distance, we show the analogous claim to be true for all positive powers if we are only interested in local stability, and for sufficiently large powers if we require global stability.

These results represent joint work with Tongseok Lim (of Purdue University's Krannert School of Management).

ABBAS MOMENI, Carleton University
Supports of extremal doubly stochastic measures and the uniqueness of the Kantorovitch optimizer

Our objective in this talk is to provide a practical necessary and nearly sufficient condition for a set to support an extremal doubly stochastic measure. We then present sufficient conditions for uniqueness of solutions of the Kantorovitch problem even though such plans may not be generally concentrated on graphs.

LEVON NURBEKYAN, UCLA
Parameter identification for chaotic dynamical systems via optimal transport

Parameter identification determines the essential system parameters required to build real-world dynamical systems by fusing crucial physical relationships and experimental data. However, the data-driven approach faces main difficulties, such as a lack of observational data, discontinuous or inconsistent time trajectories, and noisy measurements. The ill-posedness of the inverse problem comes from the chaotic divergence of the forward dynamics. Motivated by the challenges, we shift from the Lagrangian particle perspective to the state space flow field's Eulerian description. Instead of using pure time trajectories as the inference data, we treat statistics accumulated from the Direct Numerical Simulation (DNS) as the observable, whose continuous analog is the steady-state probability density function (PDF) of the corresponding Fokker–Planck equation (FPE). We reformulate the original parameter identification problem as a data-fitting, PDE-constrained optimization problem. An upwind scheme based on the finite-volume method that enforces mass conservation and positivity preserving is used to discretize the forward problem. We present theoretical regularity analysis for evaluating gradients of optimal transport costs and introduce three different formulations for efficient gradient calculation. Numerical results using the quadratic Wasserstein metric from optimal transport demonstrate this novel approach's robustness for chaotic dynamical system parameter identification.

GEOFF SCHIEBINGER, University of British Columbia
Towards a Mathematical Theory of Development

New measurement technologies like single-cell RNA sequencing are bringing 'big data' to biology. In this talk we show how optimal transport can be applied to analyze time-courses of high-dimensional gene expression data. Our ultimate goal is to develop these tools into a mathematical theory of developmental biology. We aim to answer questions like *How does a stem cell* *transform into a muscle cell, a skin cell, or a neuron? How can we reprogram a skin cell into a neuron?* We model a developing population of cells with a curve in the space of probability distributions on a high-dimensional gene expression space. We design algorithms to recover these curves from samples at various time-points and we collaborate closely with experimentalists to test these ideas on real data.

DAVE SCHNEIDER, University of Saskatchewan
Kac goes to work: Stochastic processes as probes of the architecture of plant root systems

The past decade has seen a rapid development of data-driven plant breeding strategies based on the two significant technological developments – high throughput DNA sequencing and the use of high resolution digital imaging to estimate quantitative traits related to plant architecture. Imaging above-ground structures such as shoots, leaves and flowers has developed rapidly. In contrast, below-ground structures are much more difficult to study. In part, this difficulty is associated with the lack of mathematical tools to characterize multi-scale, dendridic structures such as plant root systems. The focus of this talk, inspired by the analytical results of Kac, van den Berg and many others in the area of spectral geometry, is to describe a computational and statistical methodology that employs stochastic processes as quantitative measurement tools suitable for characterizing images of multi-scale dendritic structures. The substrate for statistical analyses in Wasserstein space are hitting distributions obtained by Monte Carlo simulation. The practical utility of this approach is demonstrated using 2D images of sorghum roots of different genetic backgrounds and grown in different environments. The work presented here is the result of collaborations with Young-Heon Kim, Hugo Lavenant, Brendan Pass, Yujie Pei and Geoff Schiebinger.

ADOLFO VARGAS-JIMENEZ, University of Alberta
Monge solutions and uniqueness in multi-marginal optimal transport via graph theory

In this talk, we will focus on the multi-marginal optimal transport problem with surplus $b(x_1, \ldots, x_m) = \sum_{\{i,j\} \in P} x_i \cdot x_j$, where $P \subseteq Q := \{\{i,j\} : i, j \in \{1, 2, ...m\}, i \neq j\}$. We associate each surplus of this type with a graph with m vertices, whose set of edges is indexed by P. We then provide a natural reformulation of the problem in a graph theory approach, and establish uniqueness and Monge solution results for two general classes of surplus functions. In particular, these classes encapsulate the Gangbo and Święch surplus and the surplus $\sum_{i=1}^{m-1} x_i \cdot x_{i+1} + x_m \cdot x_1$, whose origin lies in the time discretization of Arnold's variational interpretation of the incompressible Euler equation. This is joint work with Brendan Pass.

TING-KAM LEONARD WONG, University of Toronto
Pseudo-Riemannian geometry embeds information geometry in optimal transport

Optimal transport and information geometry both study geometric structures on spaces of probability distributions. Optimal transport characterizes the cost-minimizing movement from one distribution to another, while information geometry originates from coordinate-invariant properties of statistical inference. Their connections and applications in statistics and machine learning have started to gain more attention. We show that the pseudo-Riemannian framework of Kim and McCann, a geometric perspective on the fundamental Ma-Trudinger-Wang (MTW) condition in the regularity theory of optimal transport maps, encodes the dualistic structure of statistical manifold which is a generalization of Riemannian geometry. Some examples are given to illustrate the framework. This is joint work with Jiaowen Yang (Facebook).

KELVIN SHUANGJIAN ZHANG, École Normale Supérieure de Paris
Strong duality of the principal-agent problem with bilinear preferences and its application to characterize the solutions

The principal-agent problem is one of the central problems in microeconomics. Rochet and Choné (1998) reduced the multi-dimensional principal-agent problem with bilinear preferences to a concave maximization over the set of convex functions. We introduce a new duality and use it to characterize solutions to this problem. This is joint work with Robert J. McCann.