New perspectives on the Brunn-Minkowski theory
Nouvelles perspectives sur la théorie de Brunn-Minkowski
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YUXIN DONG, School of Mathematical Sciences, Fudan University, P. R. China
Prescribed Webster scalar curvatures on compact pseudo-Hermitian manifolds

In this talk, we will discuss the problem of prescribing Webster scalar curvatures on compact strictly pseudoconvex CR manifolds. In terms of the upper and lower solutions method and the perturbation theory of self-adjoint operators, we try to describe some sets of Webster scalar curvature functions which can be realized through pointwise CR conformal deformations and CR conformally equivalent deformations respectively from a given pseudo-Hermitian structure. This is a joint work with Yibin Ren and Weike Yu.

DMITRY FAIFMAN, Tel Aviv University
Between the Funk metric and convex geometry

The Funk metric is a non-symmetric relative of the Hilbert metric in the interior of a convex body; in some sense it interpolates between Minkowski geometry and centro-affine geometry. I will present some results concerning the Holmes-Thompson volume in Funk geometry. In particular, we will see generalizations of some well-known inequalities on the volume product in convex geometry, such as the Blaschke-Santalo inequality and the Mahler conjecture.

STEVEN HOEHNER, Longwood University
Extremal general affine surface areas

Recently, Giladi, Huang, Schütt and Werner (2020) introduced and studied the extremal L_p affine surface areas. We extend their definitions and some of their results from the setting of p-affine surface areas to the setting of general L_φ and L_ψ affine surface areas, where φ and ψ are concave and convex functions which satisfy some prescribed conditions. We also prove Blaschke-Santaló type inequalities for these new extremal affine surface areas.

HAN HONG, University of British Columbia
Index estimate for free boundary CMC surfaces and isoperimetric problem

Let Ω be a bounded domain in $\mathbb{R}^3$, the isoperimetric problem asks: what is the surface in Ω that has least area under fixed volume constraint? The existence of such surface is guaranteed and it must be free boundary constant mean curvature surfaces. We will discuss the geometry and topology of such surfaces Σ. In particular, we will give a lower bound for the Morse index:

$$\text{Index}_w(\Sigma) \geq \frac{2g + r - 4}{6}$$

where g, r are the number of genus and boundary components of the surface, respectively.

QINGZHONG HUANG, Jiaxing University
An anisotropic version of the Brezis-Van Schaftingen-Yung formula

Very recently, Brezis, Van Schaftingen and Yung established a surprising formula which connects the Marcinkiewicz quasi-norm and the Sobolev semi-norm. In this talk, I will present an anisotropic version of the Brezis-Van Schaftingen-Yung formula.

BEN LI, Ningbo University
The Existence of Extremizers of Blaschke-Santaló type Inequalities

We discuss a topological structure on families of convex functions and then apply it to show the existence of extrimizers for the functional Santaló inequality with respect to polar transform and its reverse.

JIN LI, Vienna University of Technology
Legendre transforms, Laplace transforms and valuations

Beautiful characterizations of Legendre transforms via the concept of duality were established by Artstein-Avidan and Milman [Ann. of Math. 2009]. One of the key points in their arguments is that dualities are "strong" valuations, namely, the maximum and the "minimum" of convex functions are interchanged by dualities. In this talk, I will present some characterizations of Legendre transforms via the concept of valuation and affine "invariance". Laplace transforms can be also characterized by similar properties. Hence we may say that Legendre transforms and Laplace transforms are siblings.

YOUJIANG LIN, Chongqing Technology and Business University
Affine isoperimetric inequalities

In this talk, we state some basic concepts of convex geometry analysis and give the proofs of the functional versions of the affine isoperimetric inequalities, i.e., Orlicz-Polya-Szego inequalities with respect to log-concave functions, Sobolev functions and BV functions.

ARNAUD MARSIGLIETTI, University of Florida
Concavity properties of the outer parallel volume

We investigate concavity properties of the outer parallel volume of compact sets, and discuss the relationship with important geometric inequalities such as the Brunn-Minkowski and isoperimetric inequalities.

FABIAN MUSSNIG, University of Florence
Functional Instrinsic Volumes and Hadwiger's Theorem for Convex Functions

In recent years, many results from the classical Brunn-Minkowski theory have been transfered to the functional setting. In this talk I am going to present a new class of functional intrinsic volumes for convex functions and discuss their properties. Among the many features they share with their classical counterparts is the fact that they can be characterized as continuous rigid motion invariant valuations. The results of this talk are joint work with Andrea Colesanti and Monika Ludwig.

SERGII MYROSHNYCHENKO, University of Alberta
On visual shapes and non-central sections.

Assume that Earth is made out of a transparent glass and contains a convex body K in its interior. Let K be seen as a disk from every point on the planet's surface, possibly of different radii. Can one conclude that K is a Euclidean ball? What if it is seen as an ellipse or a polygon?

We discuss related open problems, provide known and recent results that answer all of the questions above, as well as their dual counterparts for non-central sections of convex bodies.

OSCAR ORTEGA-MORENO, Vienna University of Technology
Fixed points of Minkowski valuations

In this talk, we show that for any sufficiently regular even Minkowski valuation Φ which is homogeneous and intertwines rigid motions, there exists a neighborhood of the unit ball, where balls are the only solutions to the fixed-point problem $\Phi^2 K = \alpha K$. This significantly generalizes results by Ivaki for projection bodies and suggests, via the Lutwak–Schneider class reduction technique, a new approach to Petty's conjectured projection inequality. This is joint work with Franz E. Schuster.

DMITRY RYABOGIN, Kent State University
On the chord property for the pair of convex bodies

Let K be a convex body in $\mathbb{R}^d$, $d \geq 3$, and let K_δ be its floating body for some fixed $\delta \in \left(0, \frac{\mathrm{vol}_d(K)}{2}\right)$. Assume that for all sections $K \cap H$ that are tangent to K_δ, the length of all the chords $g \subset K \cap H$ passing through the centers of mass of all $K \cap H$ is the same. Does it follow that K and K_δ are concentric Euclidean balls? We show that the answer is affirmative for bodies of revolution.

CARSTEN SCHUETT, Christian-Albrechts-Universität Kiel

KATHERYNA TATARKO, University of Alberta
Unique determination of ellipsoids by their dual volumes

Gusakova and Zaporozhets conjectured that ellipsoids in $\mathbb{R}^n$ are uniquely determined up to an isometry by their intrinsic volumes. In this talk, we will present a solution to the dual problem in all dimensions. We show that an ellipsoid is uniquely determined up to an isometry by its dual Steiner polynomial. We also discuss an alternative proof of the analogous known result of Petrov and Tarasov for classical Steiner polynomials in $\mathbb{R}^3$. This is joint work with S. Myroshnychenko and V. Yaskin.

ELISABETH WERNER, Case Western Reserve University
Blaschke-Santalo inequality for many functions and geodesic barycenters of measures

We prove a natural generalization of the Blaschke-Santalo inequality and the affine isoperimetric inequalities for many sets and many functions. We derive from it an entropy bound for the total Kantorovich cost appearing in the barycenter problem.

Based on joint work with Alexander V. Kolesnikov.

DONGMENG XI, Shanghai University
The Brunn-Minkowski type inequalities and related Minkowski problems

In this talk, we will introduce our recent works on the (dual) Brunn-Minkowski inequalities, and related Minkowski problems. These works are jointed with Yong Huang, Hailin Jin, Gangsong Leng, Yuchi Wu, Zhenkun Zhang, and Yiming Zhao.

JIE XIAO,
A Planar Minkowski Problem for the Electrostatic Capacity

This talk will address the existence-uniqueness-regularity of a planar Minkowski problem for the electrostatic capacity.

SUDAN XING, University of Alberta
On the Musielak-Orlicz-Gauss image problem

For a convex body K, its Musielak-Orlicz-Gauss image measure, denoted by $\widetilde{C}_\Theta(K, \cdot)$, involves a triple $\Theta = (G, \Psi, \lambda)$ where G and Ψ are two Musielak-Orlicz functions defined on $S^{n-1} \times (0, \infty)$ and λ is a nonzero finite Lebesgue measure on the

unit sphere S^{n-1}. Such a measure can be produced by a variational formula of $\widetilde{V}_{G,\lambda}(K)$ (the general dual volume of K with respect to λ) under the perturbations of K by the Musielak-Orlicz addition defined via the function Ψ. The Musielak-Orlicz-Gauss image problem contains many intensively studied Minkowski type problems and the recent Gauss image problem as its special cases. Under the condition that $G(\cdot,\cdot)$ is decreasing on its second variable, the existence of solutions to this problem is established. This talk is based on a joint work with Dr. Qingzhong Huang, Deping Ye and Baocheng Zhu.

VLADYSLAV YASKIN, University of Alberta
A generalization of Winternitz's theorem and its discrete version

Let K be a convex body in the plane. Cut K by a line passing through its centroid. It is a well-known result, due to Winternitz, that the areas of the resulting two pieces are at least $4/9$ times the area of K. We will discuss a discrete version of Winternitz's theorem. Joint work with Alexandra Shyntar.

NING ZHANG, Huazhong University of Science and Technology
Bodies with congruent conic sections or non-central sections in higher dimension

In this talk, we will present a recent work with Jun-Ling Li. We show that C^2 convex bodies in R^n with congruent conic sections or non-central sections will coincide. Using the continuity of multivalued function, this gives a solution to different types of Suss conjecture.

PING ZHONG, University of Wyoming
The Brown measures of free circular and multiplicative Brownian motions with nontrivial initial conditions

Free probability theory is a noncommutative version of probability theory based on free independence. Free random variables can be viewed as limit objects of certain random matrices. The limits of matrix-valued Brownian motions are described as various Brownian motions in the framework of free probability.

I will discuss briefly a joint work with CW Ho on the Brown measures of free Brownian motions with certain nontrivial initial conditions, where the subordination functions played a key role. The famous circular law (due to Ginibre, Girko, Bai, Tao, Vu and many others) states that n by n square random matrices with independent and i.i.d. entries that have mean zero and variance $1/n$ convergences to the uniform distribution on the unit disk as the size n tends to infinity. Our result for circular Brownian motion provides a density formula for the candidate of the limit operator of those random matrices perturbed by any deterministic Hermitian matrices that converge to some limit. The multiplicative version of this result extends a recent result of Driver-Hall-Kemp where they used some novel PDE methods to calculate the Brown measure of the free multiplicative Brownian motion with identity as the initial condition.

BAOCHENG ZHU, Shaanxi Normal University
The dual-polar Orlicz-Minkowski problems

We will talk about the dual-polar Orlicz-Minkowski problems: under what conditions on a nonzero finite measure μ and a continuous function $\varphi : (0,\infty) \to (0,\infty)$ there exists a convex body $K \in \mathcal{K}_o^n$ such that K is an optimizer of the following optimization problems:

$$\inf / \sup \left\{ \int_{S^{n-1}} \varphi(h_L) d\mu : L \in \mathcal{K}_o^n \right\} ?$$

Where h_L is the support function of L and S^{n-1} is the unit sphere. The solvability of the dual-polar Orlicz-Minkowski problems is discussed under different conditions. In particular, under certain conditions on φ, the existence of a solution is proved for a nonzero finite measure μ on S^{n-1} which is not concentrated on any hemisphere of S^{n-1}.