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The equations of nature and the nature of equations

Systems of N equations in N unknowns are ubiquitous in mathematical modelling. These systems, often nonlinear, are used to identify equilibria of dynamical systems in ecology, genomics, control, and many other areas. Structured systems, where the variables that are allowed to appear in each equation are pre-specified, are especially common. For modeling purposes, there is a great interest in determining circumstances under which physical solutions exist, even if the coefficients in the model equations are only approximately known.

The structure of a system of equations can be described by a directed graph G that reflects the dependence of one variable on another, and we can consider the family $\mathcal{F}(G)$ of systems that respect G. We define a solution X of $F(X) = 0$ to be robust if for each continuous F^* sufficiently close to F, a solution X^* exists. Robust solutions are those that are expected to be found in real systems. There is a useful concept in graph theory called "cycle-coverable". We show that if G is cycle-coverable, then for "almost every" $F \in \mathcal{F}(G)$ in the sense of prevalence, every solution is robust. Conversely, when G fails to be cycle-coverable, each system $F \in \mathcal{F}(G)$ has no robust solutions.

Failure to be cycle-coverable happens precisely when there is a configuration of nodes that we call a "bottleneck," a criterion that can be verified from the graph. A "bottleneck" is a direct extension of what ecologists call the Competitive Exclusion Principle, but we apply it to all structured systems.