Hopf Algebras and Related Topics
Algèbres de Hopf et sujets connexes

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MARCELO AGUIAR, Cornell University

Double monoids in duoidal categories: a brief tour and an example in geometric combinatorics

The notion of a bimonoid in a braided monoidal category is familiar to Hopf algebraists. We will discuss related but less familiar notions such as that of a double monoid. The ambient setting for the latter is that of a duoidal category. We will introduce these concepts along with examples, with the hope of reaching an example of a $(2,1)$-monoid in the category of Joyal's species which is of a beautiful geometric and combinatorial nature: it is built out of a class of polytopes called generalized permutahedra.

The talk borrows on various earlier projects done in collaboration with Swapneel Mahajan, Federico Ardila, and Jose Bastidas.

YURI BAHTURIN, Memorial University of Newfoundland

Polynomial identities of algebras with the action of Hopf algebras

In a joint paper with Felipe Yukihide Yasumura, we prove the following. Suppose we are given a Hopf algebra H and two H-simple finite-dimensional H-algebras A and B, over an algebraically closed field F. If A and B have the same polynomial H-identities then A and B are isomorphic as H-algebras.

STEFAAN CAENEPEEL, Vrije Universiteit Brussel, VUB

Frobenius Galois Rings and Corings

Brzeziński observed that the language of corings can be applied to give a unified approach to Galois theories that existed in the literature, such as Galois theory for field and commutative ring extensions with action by a finite group, Hopf-Galois theory, Galois theory for entwining structures and others. In view of duality, the theory becomes more elegant if we consider an coring that is finitely generated and projective over the base ring. However, the nicest situation is when the coring is Frobenius (that is, it is a Frobenius monoid in the category of bimodules). For instance, we do not need flatness assumptions in order to have a structure theorem. For example, the coring that is needed in order to describe Galois theory for commutative ring extensions with a finite group action (or even a finite partial group action or a partial finite groupoid action is Frobenius. The aim of this talk is develop a streamlined theory, based on the notions of Frobenius pairs and Frobenius monads in 2-categories.

STEFAN CATOIU, DePaul University

Generalized trigonometric and hyperbolic Hopf algebras

We generalize the trigonometric Hopf algebra and the less known hyperbolic Hopf algebra. One application of these is to number theory, by providing the right generalization of the Pythagorean equation and the right generalization of Fermat's Last Theorem. Another application is to Hopf algebras, namely, to the classification of finite dimensional pointed Hopf algebras.

WILLIAM CHIN, DePaul University

Coverings of pointed coalgebras and pseudocompact algebras

Every coalgebra over an algebraically closed field is Morita-Takeuchi equivalent to a pointed coalgebra. Every pointed coalgebra can be embedded in the path coalgebra of its Gabriel quiver. We describe how topological coverings of quivers can be used to produce coverings of coalgebras. For non-Galois coverings the covering coalgebras are realized as smash coproducts over

G-sets for the fundamental group G. The comodule category of the covering is then equivalent to the category of comodules graded by the G-set. The theory can be dualized to pseudocompact algebras and completed smash coproducts.

JUAN CUADRA, University of Almeria
Non-existence of integral Hopf orders for twists of simple groups of Lie type

In the papers [1] and [2] we discovered an arithmetic difference between group algebras and semisimple Hopf algebras; namely, *complex semisimple Hopf algebras may not admit integral Hopf orders.* This reveals that, unlike group algebras, Kaplansky's sixth conjecture can not be proved through the property that semisimple Hopf algebras are defined over number rings.

The families of examples for which this phenomenon occurs turn out to be simple Hopf algebras. The following question was proposed in [2]:

Let G be a finite group and Ω a non-trivial twist for $\mathbb{C}G$, arising from an abelian subgroup, such that the twisted Hopf algebra $(\mathbb{C}G)_\Omega$ is simple. Can $(\mathbb{C}G)_\Omega$ admit an integral Hopf order?

In this talk we will show that this question has a negative answer for several families of finite simple groups of Lie type, which include: special/projective special linear groups of order 2 and 3, special/projective special unitary groups of order 3, and the Suzuki groups.

The results that will be presented are part of a work in progress joint with Giovanna Carnovale and Elisabetta Masut (University of Padova, Italy).

References

[1] J. Cuadra and E. Meir, *On the existence of orders in semisimple Hopf algebras*. Trans. Amer. Math. Soc. 368 (2016), 2547-2562.

[2] ______, *Non-existence of Hopf orders for a twist of the alternating and symmetric groups*. J. London Math. Soc. (2) 100 (2019) 137-158.

JÖRG FELDVOSS, University of South Alabama
Projective Modules and Blocks of a Hopf Algebra

In this talk I will explain how certain projective modules of a finite-dimensional Hopf algebra H can be employed to estimate the number of isomorphism classes of the irreducible H-modules and the number of the blocks of H. Some of this is motivated by joint work with Salvatore Siciliano and Thomas Weigel on restricted Lie algebras.

TERRY GANNON, University of Alberta
Quantum SL2 and logarithmic vertex operator algebras

The category of modules of rational vertex operator algebras are relatively well understood. The best understood family of nonrational vertex operator algebras are the so-called triplet algebras. Their category of modules have been conjectured to coincide with the representation category of small quantum SL2 at a root of unity. My talk will review this conjecture, and explain its recent proof by Cris Negron and myself.

MIODRAG IOVANOV, University of Iowa
On Combinatorial Hopf Algebras

We introduce a combinatorial structure which generalizes graphs, multigraphs, hypergraphs, simplicial and delta complexes, colored graphs and more, which we call multi-complexes. It has a Hopf algebra structure similar to that of the Hopg algebra of graphs, where the isomorphism types of multi-complexes provide a basis and multiplication and comultiplication record assembly and disassembly combinatorial information. We find a basis of in the space of primitives of this Hopf algebra, which has combinatorial relevance in as the formulas giving the original basis in terms of primitives have non-negative integer coefficients. We give cancellation and grouping free formulas for the primitives, and also obtain the cancellation and grouping free formula for the antipode. This recovers such formulas in various other particular cases. Time permitting, we explain how some conjectures in combinatorics (specifically, graph theory) can be approached via this setup. This work is joint with Jaiung Jun.

VLADISLAV KHARCHENKO, UNAM
Quantizations as quadratic-linear Koszul algebras

The Koszul algebras arise in many areas of the modern mathematics: algebraic geometry, representation theory, noncommutative geometry, topology, number theory, theory of pseudoroots of noncommutative polynomials. We prove that in q-Weyl generators the multi-parameter Drinfeld-Jimbo quantizations of type A_n^+ and B_n^+ are quadratic-linear Koszul algebras

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ALAN KOCH, Agnes Scott College
Abelian maps, Hopf-Galois structures, and solutions to the Yang-Baxter equation

Let L/K be a nonabelian Galois extension, and let $G = \mathrm{Gal}(L/K)$. Let $\psi : G \to G$ be an endomorphism whose image is an abelian subgroup of G. We construct a K-Hopf algebra H_ψ and show that L/K is an H_ψ-Galois extension. A Hopf-Galois structure on L/K allows us to construct two skew left braces, each of which in turn gives a non-degenerate, set-theoretic solution to the Yang-Baxter equation. We explicitly describe the two skew left braces as well as the corresponding solutions.

MITJA MASTNAK, Saint Mary's University
A cohomological approach to liftings

The classification of various certain kinds of pointed Hopf algebras (and more generally Hopf Algebras with the Chevalley property) involves first describing a graded (over non-negative integers) Hopf algebra and then describing all its liftings, i.e., filtered Hopf algebras whose associated graded Hopf algebra is one of the fixed graded Hopf algebra we found in step one. In my talk I will present some ideas and recent results involved in a cohomological approach to computing liftings of a fixed graded Hopf algebra.

SUSAN MONTGOMERY,
Actions of pointed Hopf algebras on matrix rings

Let H be a finite dimensional pointed Hopf algebra with an abelian group G of group-like elements, over a field k which contains all the n^{th} roots of 1, for $n = |G|$. We determine actions of H on matrices $M_m(k)$. We obtain a complete answer when H is a Taft algebra, and partial answers for other H, in particular the Drinfeld double of the Taft algebra, for smaller matrices. Our techniques use the classification of group gradings of matrices by Bahturin, Sehgal, and Zaicev. This work is joint with Yuri Bahturin

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SIU-HUNG NG, Louisiana State University
Witt groups and signatures of modular tensor categories

In this talk, we introduce the notion of signatures of fusion categories. These signatures can be extended to Witt invariants of modular or super-modular categories. The higher central charges of any modular category can be expressed in terms of its first central charge and signature. The signatures of an infinite sequence of quantum group modular categories are proved to be $\mathbb{Z}_2$-linearly independent, which implies a conjecture of Davydov-Nikshych-Ostrik on the super-Witt group. This talk is based on a joint work with Eric Rowell, Yilong Wang and Qing Zhang.

DMITRI NIKSHYCH, University of New Hampshire
On the braid group representations coming from weakly group-theoretical fusion categories

Objects of braided tensor categories give rise to representations of braid groups. These representations are used to construct invariants of knots and links and to study topological models for quantum computing. One would like to understand a relation between these representations and the structure of the original category. We prove that braid group representations coming from weakly group-theoretical braided fusion categories have finite images. This extends the finiteness result of Etingof, Rowell, and Witherspoon for group-theoretical categories. We explicitly compute the braid group images coming from Drinfeld doubles of dihedral groups. This is a report on the joint work with Jason Green.

VICTOR OSTRIK, University of Oregon
Frobenius exact symmetric tensor categories.

I will report on a joint work in progress with K.Coulembier and P.Etingof. We give a characterization of symmetric tensor categories over fields of positive characteristic which admit an exact tensor functor to the Verlinde category; in particular we give a characterization of Tannakian categories. A crucial ingredient of this characterization is exactness of the Frobenius twist functor which mimics the Frobenius twist for representations of algebraic groups.

JULIA PLAVNIK, Indiana University
Algebraic structures in group-theoretical fusion categories

In this talk, we will present an explicit construction of Morita equivalence class representatives of indecomposable, separable algebras in group-theoretical fusion categories. This generalizes the result by Ostrik (2003) and Natale (2017) that a collection of twisted group algebras in a pointed fusion category serve as explicit Morita equivalence class representatives of indecomposable, separable algebras in such categories. We will explain the construction of our algebras and good algebraic properties that they enjoy.

This talk is based on joint work with Y. Morales, M. Müller, A. Ros Camacho, A. Tabiri, C. Walton.

PAUL TRUMAN, Keele University
Isomorphism problems for Hopf-Galois structures and skew braces

Let L/K be a finite Galois extension of fields and let $\mathcal{S}$ denote the set of Hopf-Galois structures on L/K. Each Hopf-Galois structure in $\mathcal{S}$ consists of a Hopf algebra H and a certain K-linear action of H on L; a natural way to partition $\mathcal{S}$ is to identify Hopf-Galois structures whose underlying Hopf algebras are isomorphic. On the other hand, each of these Hopf-Galois structures corresponds to a skew brace; another way to partition $\mathcal{S}$ is to identify Hopf-Galois structures whose corresponding skew braces are isomorphic. We use the interplay between these two partitions of $\mathcal{S}$ to study the Hopf algebras and skew braces involved. In particular, we show that in some cases the isomorphism class of the Hopf algebra giving a Hopf-Galois structure is determined by the corresponding skew brace. This is joint work with Alan Koch (Agnes Scott College).

HENRY TUCKER, University of California, Riverside
Frobenius-Schur indicators for some families of quadratic fusion categories

The family quadratic fusion categories provides most of the examples of "exotic" fusion categories, i.e. not coming from finite, Lie, or quantum groups. Recently, Izumi and Grossman families of modular data that are conjectured to give the modular data of Drinfel'd centers of the quadratic fusion categories in general. (In fact, it is true for all known examples.) Using this new modular data, we compute the categorical Frobenius-Schur indicators for these families, an important categorical invariant for fusion categories. Moreover, we look more closely at the relationship between indicators in the fusion category and indicators in its center. This is a preliminary report.

ROBERT UNDERWOOD, Auburn University at Montgomery
Hopf Orders in $K[C_p^3]$ in Characteristic p

Let p be a prime number, let K be a field of characteristic p that is complete with respect to a discrete valuation, and let C_p^3 denote the elementary abelian group of order p^3. We construct a large collection of Hopf orders in the K-Hopf algebra $K[C_p^3]^*$ and compute their dual Hopf orders in $K[C_p^3]$.