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Perturbation of dyadic averages

If $f:\mathbf{R}^d \to \mathbf{C}$ is locally integrable and $E \subset \mathbf{R}^d$ is bounded and measurable, with positive Lebesgue measure $|E|$, then f_E means f's average over E: $f_E := \frac{1}{|E|}\int_E f\,dt$. $\mathcal{D}$ denotes the family of dyadic cubes in $\mathbf{R}^d$. By the Lebesgue Differentiation Theorem, for a.e. $x \in \mathbf{R}^d$, $f_Q \to f(x)$ as $|Q| \to 0$, for $Q \in \mathcal{D}$ such that $x \in Q$. Suppose that, for some fixed $0 < \eta \ll 1$, and for every $Q \in \mathcal{D}$, we have an $n \times n$ real matrix $A^{(Q)}$ and a vector $y^{(Q)} \in \mathbf{R}^d$ such that: a) $\|I_d - A^{(Q)}\|_\infty < \eta$, where I_d is the identity matrix and $\|\cdot\|_\infty$ is the standard matrix norm; b) $|y^{(Q)}| \leq \eta$. For each $Q \in \mathcal{D}$ define

$$\begin{aligned} F^{(Q)}(x) &:= \chi_Q\left(A^{(Q)}(x - x_Q + \ell(Q)y^{(Q)}) + x_Q\right) \\ &=: \chi_{Q^*}(x), \end{aligned}$$

where x_Q is Q's center. We think of Q^* as a perturbation of Q resulting from a close-to-the-identity affine transformation "centered" on x_Q. The averages f_{Q^*} converge to a.e. x as $|Q| \to 0$ for $x \in Q \in \mathcal{D}$.

Elementary estimates with the Hardy-Littlewood maximal function show that, for all $s > 2$, there are constants $c(d) > 0$ and $C(d,s)$ so that if $\eta < c(d)$ then, for all $f \in L^2(\mathbf{R}^d)$,

$$\left\| \sup_{x \in Q \in \mathcal{D}} |f_Q - f_{Q^*}| \right\|_2 \leq C(d,s)\eta^{1/s}\|f\|_2.$$

We improve this to get: *There are constants $c(d) > 0$ and $C(d)$ so that if $\eta < c(d)$ then, for all $f \in L^2(\mathbf{R}^d)$,*

$$\left\| \left(\sum_{x \in Q \in \mathcal{D}} |f_Q - f_{Q^*}|^2 \right)^{1/2} \right\|_2 \leq C(d)\eta^{1/2}\|f\|_2.$$