Geometric Analysis
Analyse géométrique
(Org: **Robert Haslhofer** (Toronto) and/et **Aaron Naber** (Northwestern))

SALIM DEAIBES, University of Toronto
Minimal Two-Spheres in Three-Spheres with an Arbitrary Metric

In this talk, I will explain how we prove that every Riemannian three-sphere contains at least two embedded minimal two-spheres or admits an optimal foliation by two-spheres; I will also explain why we are unable to conclude the existence of two solutions in general. This improves results of White and Haslhofer-Ketover where the existence of at least two solutions has been established under the additional assumption that the metric has positive Ricci curvature or is generic, respectively.

AILANA FRASER, University of British Columbia
Continuity of eigenvalues under degenerations

We will discuss the question of the degenerations of Riemannian manifolds under which the first k Steklov eigenvalues are continuous. This question is important when one attempts to construct metrics which optimize an eigenvalue. As an application we describe several results concerning the optimization of higher Steklov eigenvalues. This talk includes joint work with R. Schoen and joint work with P. Sargent.

VITALI KAPOVITCH, University of Toronto
Mixed curvature almost flat manifolds

A celebrated theorem of Gromov says that given $n > 1$ there is an $\epsilon(n) > 0$ such that if a closed Riemannian manifold M^n satisfies $-\epsilon < sec_M < \epsilon, diam(M) < 1$ then M is diffeomorphic to an infranilmanifold. I will show that the lower sectional curvature bound in Gromov's theorem can be weakened to the lower Bakry-Emery Ricci curvature bound. I will also discuss the relation of this result to the study of manifolds with Ricci curvature bounded below.

SPIRO KARIGIANNIS, University of Waterloo
Towards higher dimensional Gromov compactness in G_2 and Spin(7) manifolds

Let (M, ω) be a compact symplectic manifold with a compatible almost complex structure J. We can study the space of J-holomorphic maps $u \colon \Sigma \to (M, J)$ from a compact Riemann surface into M. By "compactifying" the space of such maps, one can obtain powerful global symplectic invariants of M. This requires understanding the ways in which sequences of such maps can develop singularities. Crucial ingredients are conformal invariance and an energy identity, which lead to to a plethora of analytic consequences, including: (i) a mean value inequality, (ii) interior regularity, (iii) a removable singularity theorem, (iv) an energy gap, and (v) compactness modulo bubbling.

Riemannian manifolds with closed G_2 or $\mathrm{Spin}(7)$ structures share many similar properties to such almost Kahler manifolds. In particular, they admit analogues of J-holomorphic curves, called associative and Cayley submanifolds, respectively, which are calibrated and hence homologically volume-minimizing. A programme initiated by Donaldson-Thomas-Segal aims to construct similar such "counting invariants" in these cases. In 2011, an overlooked preprint of Aaron Smith demonstrated that such submanifolds can be exhibited as images of a class of maps $u \colon \Sigma \to M$ satisfying a conformally invariant first order nonlinear PDE analogous to the Cauchy-Riemann equation, which admits an energy identity involving the integral of higher powers of the pointwise norm $|du|$. I will discuss joint work (to appear in Asian J. Math.) with Da Rong Cheng (Waterloo) and Jesse Madnick (NCTS/NTU) in which we establish the analogous analytic results of (i)-(v) in this setting. arXiv:1909.03512

CHRISTOPHER KENNEDY, University of Toronto
A Bochner Formula on Path Space for the Ricci Flow

Aaron Naber (Northwestern) and Robert Haslhofer (Toronto) have characterized solutions of the Einstein equation $\mathrm{Rc}(g) = \lambda g$ in terms of both sharp gradient estimates for Brownian motion and a Bochner formula on elliptic path space PM. They also successfully characterized solutions of the Ricci flow $\partial_t g = -2\mathrm{Rc}(g)$ in terms of an infinite-dimensional gradient estimate on parabolic path space $P\mathcal{M}$ of space-time $\mathcal{M} = M \times [0, T]$.

In this talk, we shall generalize the classical Bochner formula for the heat flow on evolving manifolds $(M, g_t)_{t\in[0,T]}$ to an infinite-dimensional Bochner formula for martingales, thus proving the parabolic counterpart of recent results in the elliptic setting as well as characterizing solutions of the Ricci flow in terms of Bochner inequalities on parabolic path space. Time-permitting, we shall also discuss gradient and Hessian estimates for martingales on parabolic path space as well as a condensed proof of previous characterizations of the Ricci flow.

SIYUAN LU, McMaster University
Rigidity of Riemannian Penrose inequality with corners and its implications

Motivated by the rigidity case in the localized Riemannian Penrose inequality, we show that suitable singular metrics attaining the optimal value in the Riemannian Penrose inequality is necessarily smooth in properly specified coordinates. If applied to hypersurfaces enclosing the horizon in a spatial Schwarzschild manifold, the result gives the rigidity of isometric hypersurfaces with the same mean curvature. This is a joint work with Pengzi Miao.

ANTHONY MCCORMICK, Northwestern University
Ladder Asymptotics on Stationary Spacetimes

The space of solutions to the wave equation on a principal bundle over a stationary spacetime decomposes in terms of isotypic representations of the structure group. We present a trace formula for the unitary time evolution operator when restricted to a ladder of representations and analyze the corresponding limit of large quantum numbers, providing a common extension of some results of Guillemin-Uribe and Strohmaier-Zelditch.

JEFF STREETS, UC Irvine
Generalized Ricci Flow

The generalized Ricci flow is a geometric evolution equation coupling the classic Ricci flow to equations for 'torsion,' and arises independently in mathematical physics, generalized geometry, and complex geometry. In this talk I will survey recent progress on this equation including new global existence results for the flow, and classification and rigidity results for generalized Ricci solitons.

JEROME VETOIS, McGill University
Existence results for the higher-order Q-curvature equation

In this talk, we will discuss the problem of prescribing the Q-curvature of order $2k$ on a closed Riemannian manifold of dimension $n > 2k$, where k is an integer. This amounts to solving a nonlinear elliptic PDE involving a $2k$-th order operator called the Graham-Jenne-Mason-Sparling (GJMS) operator. I will present new existence results for this problem under assumptions of coercivity of the operator and positivity of the Green's function, which are satisfied for instance when the manifold is Einstein. An additional positive mass assumption is also required in the case of small dimensions $2k + 1 \leq n \leq 2k + 3$ and locally conformally flat manifolds. This is a joint work with Saikat Mazumdar (Indian Institute of Technology Bombay).