Ergodic Theory, Dynamical Systems, Fractals and Applications
La théorie ergodique, les systèmes dynamiques, les fractales et les applications
(Org: **Jacopo de Simoi** (Toronto) and/et **Shafiqul Islam** (PEI))

ARNO BERGER, University of Alberta
Digits and dynamics - an update

This talk presents a selection of results concerning the distribution of significant digits and significands, with an emphasis on data generated by dynamical processes (deterministic or random, linear or nonlinear, discrete or continuous). Several intriguing questions emerge naturally, pertaining to analysis, probability and number theory. (Based on joint work with G. Eshun, S. Evans, and T. Hill.)

ILLIA BINDER, University of Toronto
Critical Interfaces and SLE: the rate of convergence

I will describe the general framework for establishing the polynomial rate of convergence of critical lattice interfaces to SLE curves. I will also discuss the applications of this framework to various planar models of Statistical Physics, such as Ising, Harmonic Explorer, and Critical Percolation. The talk is based on joint works with Larissa Richards (University of Toronto) and Dmitry Chelkak (École normale supérieure).

CHRIS BOSE, cbose@uvic.ca
Bounded distortion for random maps sampled across large parameter intervals.

Recently, with Anthony Quas (Victoria) and Matteo Tanzi (NYU) we have established bounded distortion estimates for randomized intermittent Liverani-Saussol-Vaienti (LSV) maps sampled across the full parameter range. Obtaining bounded distortion is a key step in deriving decay of correlation asymptotics for iterated maps and all kinds of random maps. We will briefly outline how this works and discuss alternatives to bounded distortion that can be applied in some settings.

PEYMAN ESLAMI, University of Roma Tor vergata
Exponential mixing for skew products with a holder roof function

Consider the skew product $F(x,y) = (f(x), y + \tau(x))$ on $S^1 \times S^1$, where $f \in C^{1+}$, as the base map, is (piecewise) expanding. When τ, as the roof function, has C^{1+} regularity (and not locally constant) it is known that F mixes exponentially. However, in applications of this problem one is usually faced with g being only Holder continuous with exponent strictly less than one. I will discuss the exponential mixing of F in this more general situation.

CHRISTOPHER ESSEX, Applied Mathematics/Mathematics, UWO
The Entropy Production Paradox and Fractional Master Equations

The entropy production paradox concerns the unexpected and robust increase of entropy production rates as one moves away from the (irreversible) diffusion equation to approach the (reversible) wave equation. This unexpected behaviour was discovered while studying fractional diffusion equations meant to capture anomalous super diffusion. It has shown up robustly on different domains for distinct evolution equations with rather different probability density functions, all of which exhibit what we called pseudo propagation. Broadening this investigation to fractional master equations, on a bounded domain, leads to the paradox again, but only as a transient, which ultimately relaxes to classical expectations, providing insight into the original paradox and the nature of irreversibility.

KASUN FERNANDO, University of Toronto
The Bootstrap for Chaotic Dynamical Systems

Parameter estimation problems in dynamical systems arise naturally in many applications in machine learning, physics, biology, econometrics, and engineering. However, there are several standard statistical techniques that have not yet been implemented in the setting of dynamical systems. One such technique is the Bootstrap which is a widely-used resampling technique assigning measures of accuracy to sample estimates. In this talk, we introduce the Bootstrap for (exponentially mixing) dynamical systems. To establish its asymptotic accuracy, we establish the *continuous* Edgeworth expansions for dynamical systems. We also verify our theoretical results through simulations. This is joint work with Nan Zou (Macquarie University).

PAWEL GORA, Concordia University
Periodic Islands for 2-dim Maps

We consider a two dimensional map

$$G(x, y) = (y, f(\alpha y + (1 - \alpha)x),$$

where $f(t) = 1 - 2|t - 1/2|$ is the tent map or $f(t) = 4t(1 - t)$ is the logistic map, and $0 \leq \alpha \leq 1$ is a parameter.

For specific values of α the connected support of the absolutely continuous invariant measure (its existence is an unproven conjecture) disintegrates into a number of separate "islands" which still seem to support the acim. The map moves the islands periodically giving an example of a "weak chaos", a seemingly periodic motion which actually is chaotic.

We present a number of examples of periodic islands for different values of α. No theoretical results are presented, we only show computer generated images.

PATRICK INGRAM, York University
Critical orbits of certain endomorphisms of projective space

Post-critically finite (PCF) rational functions of one variable, those whose critical points all have finite forward orbit, are a natural class to consider in holomorphic dynamics. It follows from work of Thurston and McMullen that these functions do not occur in algebraic families, except for the Lattes examples. In higher dimension, less is known. This talk will use some tools from arithmetic geometry to say something about this problem for endomorphisms of $\mathbb{P}^N$ ramified along $N + 1$ hyperplanes (in some sense the simplest ramification one could have).

FRANKLIN MENDIVIL, Acadia University
Sizes of rearrangements of linear Cantor sets

Each compact subset of $[0, 1]$ is defined by its (countable) collection of complementary gaps. The collection of all of the lengths of these gaps encodes a great deal of information about the geometry of the set (in particular various dimensions). A "rearrangement" of a set has the same collection of gap lengths (but with a different ordering). In this talk we will give a brief survey of results about the "size" (box-counting, packing, Hausdorff, and Assouad dimensions) of rearrangements of a Cantor set. (Joint work with Ignacio Garcia, Kathryn Hare, and Leandro Zuberman)

ISRAEL NCUBE, Alabama A & M University
Distributional statistical properties and the stability of an equilibrium of a delayed symmetric network

We consider a certain class of Cohen-Hopfield-Grossberg symmetric networks characterised by multiple distributed time delays. We establish explicit analytical results on some ramifications of distributional heavy-tailedness on the stability boundary, in an appropriate parameter space, of an equilibrium of such a network. The premise of the approach adopted here is that very limited information about the time delays is available.

ANTHONY QUAS, University of Victoria
Random compositions of Blaschke products

We consider random compositions of Blaschke products and look at the corresponding Perron-Frobenius operators acting on densities. We study the Lyapunov exponents of these systems, and give a precise description of the Oseledets spectrum. (Joint work with Cecilia González-Tokman).

CHRISTIANE ROUSSEAU, Université de Montréal
Polynomial vector fields on $\mathbb{C}$

The study of polynomial vector fields $dz/dt = P(z)$ on $\mathbb{C}$ with complex methods was initiated by Douady, Estrada and Sentenac, who introduced a combinatorial invariant and an analytic invariant for generic vector fields. Together with Arnaud Chéritat, we introduced a new invariant, the periodgon, for polynomial vector fields of the form $dz/dt = z^k - \varepsilon$. In joint work with Martin Klimes, we generalized the periodgon for polynomial vector fields on $\mathbb{C}$. The periodgon is uniquely defined for "generic" vector fields, but genericity here is different from the notion introduced by Douady, Estrada and Sentenac. Furthermore, when the vector field varies, the deformation of the periodgon allows an immediate derivation of the bifurcation diagram. The study of polynomial vector fields on $\mathbb{C}$ was motivated by their importance in the study of unfoldings of parabolic points of diffeomorphisms and other similar questions.

MATTEO TANZI, CIMS, New York University
Random-like properties of chaotic forcing

We prove that skew systems with a sufficiently expanding base have "approximate" statistical properties similar to random ergodic Markov chains. For example, they exhibit approximate exponential decay of correlations, meaning that the exponential rate is observed modulo a controlled error. The fiber maps are only assumed to be Lipschitz regular and to depend on the base in a way that guarantees diffusive behaviour on the vertical component. The assumptions do not imply an hyperbolic picture and one cannot rely on the spectral properties of the transfer operators involved. The approximate nature of the result is the inevitable price one pays for having so mild assumptions on the dynamics on the vertical component. The error in the approximation is shown to go to zero when the expansion of the base tends to infinity.

SHIROU WANG, University of Alberta
A coupling approach in the computation of geometric ergodicity for stochastic dynamics

This talk introduces a probabilistic approach to numerically compute geometric convergence rates in discrete or continuous stochastic systems. Choosing appropriate coupling mechanisms and combining them together, this approach works well in many settings, especially in high-dimensions. It is particularly observed that the rate of geometric ergodicity of a randomly perturbed system can, to some extent, reveal the degree of chaoticity of the unperturbed system. This talk is based on a joint work with Yao Li.

KOUJI YANO, Kyoto University
Arcsine law for a piecewise linear random map

We construct a random interval map by choosing randomly two piecewise linear maps whose orbits converge to 0 or 1, and show that it obeys Thaler–Zweimüller's arcsine law. This talk is based on a joint work with Genji Hata and Toru Sera.

JAMES YORKE, Univ of Maryland College Park
Robust solutions in systems of equations

Joint work with Sana Jahedi and Tim Sauer.

We begin by asking what would we like (really smart) high-school students to know about systems of equations. A structured system of equations $F(x) = c$ where $F : \mathbb{R}^N \to \mathbb{R}^M$ is a system of M equations in which it is specified which of the N variables are allowed to appear in each equation. They are ubiquitous in mathematical modeling. Our goal in this article is to describe the global properties of solutions for structured systems of C^∞ functions. We describe general properties of solutions that follow from the system structure rather than from the particular details of the system. An important question when modeling natural systems is whether solutions are robust to small changes in the model. To address this question, we say F is "flat" if there exists an integer k such that for almost every x_0, the set $F^{-1}F(x_0)$ of solutions x of $F(x) = F(x_0)$ is a k-dimensional manifold. When $k = 0$, the solutions are isolated points. Systems of polynomials are examples of flat functions. But there are C^∞ functions F that are not flat, even when $N = M = 1$. We state conditions on vector spaces of C^∞ functions that imply that "almost every" (in the sense of prevalence) F in the vector space is k-flat for some k, and we show that if $k = M - N$ almost every F has the property that almost every x is a robust solution of $F(x) = c$ for some c. Then x is robust-to-small-changes in c and F.

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