**Recent Advances in Complex and Harmonic Analysis**
**Progrès récents dans l'analyse harmonique complexe**
(Org: **Ilia Binder** (Toronto), **Damir Kinzebulatov** (Laval) and/et **Javad Mashreghi** (Laval))

**EUGENE BILOKOPYTOV**, University of Alberta
*Multiplier Algebras, big and small*

In this talk we consider multiplier algebras of Banach spaces of continuous and analytic functions. In particular, conditions which guarantee that such a multiplier algebra is big (i.e. non-separable) is presented. We also discuss some situations when a Banach space of functions has no non-constant multipliers. In order to construct an example of such a space over an arbitrary separable metric space we use a generalization of a result by Mashreghi and Ransford about realization of every separable Banach space as a Banach space of analytic functions.

**LUDOVICK BOUTHAT**, Université Laval
*Some results about infinite L-matrices*

We know that any linear application from $\mathbb{C}^n$ to $\mathbb{C}^n$ can be described with an $n \times n$ square matrix. The space $\ell^2$ of square-summable sequences indexed by the natural numbers is a generalization of $\mathbb{C}^n$ to infinite dimension. We find that the operators, in the case of $\ell^2$, can be described by infinite matrices. However, not all infinite matrices gives us an operator on $\ell^2$. It is natural to wonder which infinite matrices are a representation of an operator on $\ell^2$, and what is their norm. Because of their applications in the problem of the caracterisation of the multipliers in the weighted Dirichlet spaces, we restrict ourselves to the case of infinite $L$-matrices. An infinite positive $L$-matrix is an infinite matrix which is defined by a sequence $(a_n)_{n\geq 0}$ of positive real numbers and which is of the form

$$A = \begin{pmatrix} a_0 & a_1 & a_2 & a_3 & \cdots \\ a_1 & a_1 & a_2 & a_3 & \cdots \\ a_2 & a_2 & a_2 & a_3 & \cdots \\ a_3 & a_3 & a_3 & a_3 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

We present some conditions on the sequence $(a_n)_{n\geq 0}$ for $A$ to be an operator on $\ell^2$ and we present a particular set of $L$-matrices for which we are able to exactly determine the norm. We also show some new results about $L$-matrices with lacunary coefficients.

**ALEXANDER BRUDNYI**, Calgary
*ON NONLINEAR RUDIN-CARLESON TYPE THEOREMS*

Let $\bar{\mathbb{D}} \subset \mathbb{C}$ be the closed unit disk and $\mathbb{T} \subset \bar{\mathbb{D}}$ be the unit circle. The classical Rudin-Carleson theorem asserts that if $S \subset \mathbb{T}$ is a closed subset of Lebesgue measure zero, then for every complex continuous function $f$ on $S$ there exists a continuous function $g$ on $\bar{\mathbb{D}}$ holomorphic in its interior $\mathbb{D}$ such that $g|_S = f$ and $\max_{\bar{\mathbb{D}}} |g| \leq \max_S |f|$.

In the talk, I present analogs of this interpolation theorem for continuous maps into complex manifolds.

**MAXIM BURKE**, University of Prince Edward Island
*Analytic order-isomorphisms of countable dense subsets of the unit circle*

For functions in $C^k(\mathbb{R})$ which commute with a translation, we prove a theorem on approximation by entire functions which commute with the same translation, with a requirement that the values of the entire function and its derivatives on a specified countable set belong to specified dense sets. Using this theorem, we show that if $A$ and $B$ are countable dense subsets of the unit circle $T \subseteq \mathbb{C}$ with $1 \notin A$, $1 \notin B$, then there is an analytic function $h \colon \mathbb{C} \setminus \{0\} \to \mathbb{C}$ that restricts to an order isomorphism of the arc $T \setminus \{1\}$ onto itself and satisfies $h(A) = B$ and $h'(z) \neq 0$ when $z \in T$. This answers a question of P. M. Gauthier.

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**ALMAZ BUTAEV**, University of Calgary
*On locally uniform domains in $\mathbb{R}^n$*

I will talk about different definitions of locally uniform domains $\Omega \subset \mathbb{R}^n$. Specifically, we will be interested in the characterization of locally uniform domains in terms of the quasi-hyperbolic metric and their role as extension domains for Goldberg's bmo($\Omega$) space. This is joint work with Galia Dafni (Concordia University).

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**GALIA DAFNI**, galia.dafni@concordia.ca
*Vanishing mean oscillation*

Sarason (1975) characterized the closure of the uniformly continuous functions in $\mathrm{BMO}(\mathbb{R})$ by the uniform vanishing of the mean oscillation (VMO) over intervals, as the size of the intervals shrinks to zero. Variations of VMO exist in the literature under the same and different notations. In joint work with Almut Burchard (Toronto) and Ryan Gibara (Laval), we consider VMO defined using a basis of open sets in $\mathbb{R}^n$ and study the continuity of rearrangements on this space. In joint work with Almaz Butaev (Calgary), looking at the nonhomogenous BMO space (Goldberg's bmo) on a domain $\Omega \subset \mathbb{R}^n$, we formulate conditions determining "vanishing at the boundary" and "vanishing at infinity", and obtain approximation and extension results for functions satisfying these conditions when $\Omega$ is an $(\epsilon, \delta)$ domain.

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**PAUL GAUTHIER**, Université de Montréal
*A characterization of non-tangential cluster sets for holomorphic functions $f : D \to D$.*

For a holomorphic function in the unit disc, denote the non-tangential (=angular) cluster set as $C_{NT}(f, 1)$. Harald Woracek asked for a description (other than the definition) of sets $A$, for which $A = C_{NT}(f, 1)$, for some holomorphic function in the unit disc and bounded by $1$. We characterize such sets as the union of a countable increasing sequence of continua in the closed disc.

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**ADI GLUCKSAM**, University of Toronto
*Integral mean spectrum and its complex extension- a survey*

In a celebrated paper from 1998 N. Makarov related the integral mean spectrum and the packing spectrum. In this talk I will discuss the complex version of the integral means spectrum, and present similar known relations for the complex case.

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**WENBO LI**, University of Toronto
*Quasisymmetric Embeddability of Weak Tangents*

A weak tangent of a metric space is a "blown up" space (in the sense of pointed Gromov-Hausdorff limit) near a point. In this talk, we study the quasisymmetric embeddability of weak tangents of metric spaces.

We first show that quasisymmetric embeddability is hereditary, i.e., if X can be quasisymmetrically embedded into Y, then every weak tangent of X can be quasisymmetrically embedded into some weak tangent of Y, given that X is proper and doubling. However, the converse implication is not true in general; we will illustrate this with a counterexample. In special situations, we are able to show that the embeddability of weak tangents implies the global or local embeddability of the ambient space. Finally, we apply our results on limit sets of Kleinian groups and visual spheres of expanding Thurston maps.

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**KODJO RAPHAËL MADOU**, Université Laval
*On admissible singular drifts of symmetric $\alpha$-stable process*

We consider the problem of existence of a (unique) weak solution to the SDE describing symmetric $\alpha$-stable process with a locally unbounded drift $b : \mathbb{R}^d \to \mathbb{R}^d$, $d \geq 3$, $1 < \alpha < 2$. In this talk, $b$ belongs to the class of weakly form-bounded vector fields, the class providing the $L^2$ theory of the non-local operator behind the SDE, i.e. $(-\Delta)^{\frac{\alpha}{2}} + b \cdot \nabla$. It contains as proper sub-classes other classes of singular vector fields studied in the literature in connection with this operator, such as the Kato class, the weak $L^{d/(\alpha-1)}$ class and the Campanato-Morrey class (in general, such $b$ makes invalid the standard heat kernel estimates in terms of the heat kernel of the fractional Laplacian). We show that the operator $(-\Delta)^{\frac{\alpha}{2}} + b \cdot \nabla$ with weakly form-bounded $b$ admits a realization as (minus) Feller generator, and that the probability measures determined by the Feller semigroup (uniquely in appropriate sense) admit description as weak solutions to the corresponding SDE. The proof is based on detailed regularity theory of $(-\Delta)^{\frac{\alpha}{2}} + b \cdot \nabla$ in $L^p$, $p > d - \alpha + 1$.

The talk is based on joint work with Damir Kinzebulatov (Université Laval).

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**JAVAD MASHREGHI**, Laval University
*Approximation by modified Taylor polynomials*

It is known that the sequence of Taylor polynomials may diverge in the local Dorochlet spaces. However, the sequence of Fejer means is a good remedy and it converges to the initial function in the norm. Another possibility is to modify the last term of Taylor polynomials and create a convergent sequence. We study this phenomenon as an orthogonal projection to the subspace of polynomials of degree at most $n$.

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**FRÉDÉRIC MORNEAU-GUÉRIN**, Université TELUQ
*Inégalités du type Young pour les espaces Lp(G,w)*

Au cours de cet exposé, nous présenterons deux généralisations de l'inégalité de Young pour les espaces pondérés de fonctions de p-ième puissance intégrable définies sur un groupe localement compact.

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**MAËVA OSTERMANN**, Université Laval
*Une approche abstraite de la conjecture de Crouzeix*

En 2004, Crouzeix a conjecturé que l'inégalité $\|P(T)\| \leq 2\|P\|_{W(T)}$ tient pour toute matrice $T$ et tout polynôme $P$. Récemment, Crouzeix et Palencia ont montré que cette inégalité tient avec $1 + \sqrt{2}$ à la place de $2$. En partant de leur résultat, je proposerai dans cet exposé une approche abstraite de cette conjecture.

Travail conjoint avec Thomas Ransford.

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**PIERRE-OLIVIER PARISÉ**, Université Laval
*Power-series methods in de Branges-Rovnyak spaces*

In this talk, I will introduce the logarithmic power-series method, which applies to the sequence of Taylor partial sums of a holomorphic function in the unit disk $\mathbb{D}$. I will show that there exist a de Branges-Rovnyak space $\mathcal{H}(b)$, a function $f \in \mathcal{H}(b)$ such that the polynomials are dense in $\mathcal{H}(b)$, but the Taylor series of the function $f$ is not summable with respect to the logarithmic power-series method. I will also discuss an abstract result in operator theory showing that if one regular summability method includes another for scalar sequences, then it automatically does so for certain Banach-space-valued sequences too. Lastly, I will present consequences of this result to summability in $\mathcal{H}(b)$ with respect to other power-series methods.

Joint work with Javad Mashreghi and Thomas Ransford.

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**MAREK PTAK**, University of Agriculture, Kraków, Poland
*Generalized multipliers for left-invertible analytic operators*

A left-invertible analytic operator $T$ can be seen as a multiplication operator by an independent variable on a space of analytic functions with values in kernel of the adjoint $\ker T^*$ of the given operator $T$. We define generalized multipliers for $T$ as "analytic" sequences, whose coefficients are bounded operators on $\ker T^*$. The generalized multipliers form a Banach algebra and characterize the commutant of the left-invertible analytic operator.

Joint work with Piotr Dymek and Artur Płaneta.

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**THOMAS RANSFORD**, Université Laval
*Decay of singular inner functions*

A singular inner function is a holomorphic function on the unit disk of the form

$$S(z) := \exp\Big(-\int \frac{e^{it}+z}{e^{it}-z}\,d\mu(t)\Big),$$

where $\mu$ is a finite positive Borel measure on the unit circle that is singular with respect to Lebesgue measure. A well-known and important property of such functions is that $\lim_{r\to 1^-} S(re^{i\theta}) = 0$ $\mu$-almost everywhere on the unit circle. In this talk I shall discuss the rate of convergence to zero.

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**WILLIAM ROSS**, University of Richmond
*The Smirnov class of de Branges–Rovnyak spaces*

In this joint work with Emmanuel Fricain and Andreas Hartmann, we show that every function in certain de Branges–Rovnyak spaces can be written as the quotient of two multipliers of these spaces. These types of results hold for many other analytic function spaces such as the Hardy and Dirichlet spaces.

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**ERIC SCHIPPERS**, University of Manitoba
*Transmission of harmonic functions of finite Dirichlet norm*

Consider a Jordan curve in the Riemann sphere. A harmonic function with finite Dirichlet norm on the interior of the curve has non-tangential limits in a certain sense except on a negligible set. If the Jordan curve is sufficiently regular, these are also the boundary values of a harmonic function of bounded Dirichlet norm on the exterior of the curve. We call this harmonic function on the exterior the "transmission" of the original harmonic function. The transmission operator exists and is bounded if and only if the curve is a quasicircle. We will discuss transmission and related results for the Cauchy and Grunsky operators, as well as integral operators of Schiffer.

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**RASUL SHAFIKOV**, University of Western Ontario
*Local polynomial convexity of Levi-flat hypersurfaces*

I will discuss the problem of local polynomial convexity of Levi-flat hypersurfaces near singular points.

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**ALAN SOLA**, Stockholm University
*Stable polynomials and bounded rational functions of several variables*

Reporting on joint work in progress with Kelly Bickel (Bucknell), Greg Knese (Washington University), and James Pascoe (Florida) I will discuss several problems related to polynomials in several variables having no zeros in a prescribed domain in $\mathbb{C}^n$ and to rational functions having such polynomials as their denominators.

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**ALEX STOKOLOS**, Georgia Southern University
*On univalent polynomials*

We present a method which seems to produce univalent in $\mathbb{D}$ polynomials as well as T-symmetrized version of them. This is a joint work with Dmitriy Dmitrishin and Mihai Tohaneanu.

**IGNACIO URIARTE-TUERO**, University of Toronto

*The Krzyz conjecture revisited*

The conjecture of Krzyż, concerning the largest possible value of the Taylor coefficient $a_n$ $(n \geq 1)$ of a non-vanishing analytic function from the unit disk into the unit disk, has been open since 1968 in spite of the information available on the structure of extremal functions.

The purpose of this talk is to report on partial progress regarding the conjecture. We collect various conditions that the coefficients of an extremal function (and also the zeros of some polynomials associated with it) must satisfy and show that each one of these properties is equivalent to the conjecture itself.

This improves or complements a number of earlier findings by other authors and may hopefully provide several possible starting points for attempts at proving the conjecture.

**WILLIAM VERREAULT**, Université Laval

*Nonlinear Oscillatory Expansions of Analytic functions*

We extend results of Coifman, T. Qian et al. on the Blaschke unwinding series expansion of an entire function $f$, a nonlinear analogue of Fourier series with a wide range of practical applications. To do this, we consider an unwinding of $f$ by elements in the closed unit ball of $H^\infty$. We also present convergence theorems in $H^p$ for our unwinding series.