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Analytic order-isomorphisms of countable dense subsets of the unit circle

For functions in $C^k(\mathbb{R})$ which commute with a translation, we prove a theorem on approximation by entire functions which commute with the same translation, with a requirement that the values of the entire function and its derivatives on a specified countable set belong to specified dense sets. Using this theorem, we show that if A and B are countable dense subsets of the unit circle $T \subseteq \mathbb{C}$ with $1 \notin A$, $1 \notin B$, then there is an analytic function $h: \mathbb{C} \setminus \{0\} \rightarrow \mathbb{C}$ that restricts to an order isomorphism of the arc $T \setminus \{1\}$ onto itself and satisfies $h(A) = B$ and $h'(z) \neq 0$ when $z \in T$. This answers a question of P. M. Gauthier.