Algebraic number theory
Théorie algébrique des nombres
(Org: **Alex Bartel** (Glasgow) and/et **Antonio Lei** (Laval))

BENJAMIN BREEN, Clemson University
Heuristics for narrow class groups and unit signatures of abelian number fields with odd degree.

We present heuristics for the behavior of a certain collection of ray class groups that arise when studying the narrow class group. These heuristics allow us to make predictions for unit signature ranks and the 2-torsion of narrow class groups. We demonstrate our predictions and provide computational support for abelian extensions of degree n = 3,5,7. This is joint work with Ila Varma and John Voight.

FRANCESC CASTELLA, University of California Santa Barbara
On a conjecture of Darmon–Rotger in the adjoint CM case

Let E be an elliptic curve over $\mathbf{Q}$ such that $L(E,s)$ has sign $+1$ and vanishes at $s=1$, and let $p>3$ be a prime of good ordinary reduction for E. A construction of Darmon–Rotger attaches to E, and an auxiliary weight one cuspidal eigenform g such that $L(E,\mathrm{ad}^0(g),1)\neq 0$, a Selmer class $\kappa_p(E,g,g^*)\in\mathrm{Sel}(\mathbf{Q},V_pE)$. They conjectured that the following are equivalent: (1) $\kappa_p(E,g,g^*)\neq 0$, (2) $\dim_{\mathbf{Q}_p}\mathrm{Sel}(\mathbf{Q},V_pE)=2$.

In this talk I will outline a proof of Darmon–Rotger's conjecture when g has CM and $\mathrm{Sha}(E/\mathbf{Q})[p^\infty]<\infty$ (and some mild additional hypotheses). If time permits, I'll also say a few words about the ongoing extension of these results to the case of supersingular primes p. Based on joint work with Ming-Lun Hsieh.

HARRIS DANIELS, Amherst College
This talk is Galois-entangled with Álvaro Lozano-Robledo's talk

Let E be an elliptic curve defined over $\mathbb{Q}$. The adelic Galois representation attached to E (this object will be defined during the talk) captures all sorts of interesting information about the arithmetic of the points on $E(\overline{\mathbb{Q}})$, including data about the torsion subgroup, isogenies, and other finer invariants of the curve and its isogeny class. In this talk, and in Álvaro Lozano-Robledo's talk, we will give a summary of recent results towards the classification (up to isomorphism) of the possible adelic Galois representations that arise from elliptic curves over $\mathbb{Q}$. We will present some recent results of the authors and their collaborators (Álvaro Lozano-Robledo, Jackson Morrow) about the ways in which the division fields of an elliptic curve can be entangled. Our talks will be mostly self-contained, but very much related... entangled, if you will.

CHANTAL DAVID, Concordia University
One-Level density for cubic characters over the Eisenstein field

We show that the one-level density for L-functions associated with the cubic residue symbols χ_n, with $n\in Z[\omega]$ square-free, satisfies the Katz-Sarnak conjecture for all test functions whose Fourier transforms are supported in $(-13/11,13/11)$, under GRH. This is the first result extending the support outside the trivial range $(-1,1)$ for a family of cubic L-functions. This implies that a positive proportion of the L-functions associated with these characters do not vanish at the central point $s=1/2$. A key ingredient is a bound on an average of generalized cubic Gauss sums at prime arguments, whose proof is based on the work of Heath-Brown and Patterson.

Joint work with Ahmet M. Guloglu.

JULIE DESJARDINS, University of Toronto
Density of rational points on a family of del Pezzo surface of degree 1

Let X be an algebraic variety over a number field k. We want to study the set of k-rational points X(k). For example, is X(k) empty? If not, is it dense with respect to the Zariski topology? Del Pezzo surfaces are classified by their degrees d, an integer between 1 and 9. Manin and various authors proved that for all del Pezzo surfaces of degree >1 is dense provided that the surface has a k-rational point (that lies outside a specific subset of the surface for d=2). For d=1, the del Pezzo surface always has a rational point. However, we don't know it the set of rational points is Zariski-dense. In this talk, I present a result, joint with Rosa Winter, in which we prove the density of rational points for a specific family of del Pezzo surfaces of degree 1 over k.

MICHELE FORNEA, Columbia University
Plectic Stark-Heegner points

Heegner points play a pivotal role in our understanding of the arithmetic of modular elliptic curves. They control the Mordell-Weil group of elliptic curves of rank 1, and they arise as CM points on Shimura curves. The work of Bertolini, Darmon and their schools has shown that p-adic methods can be successfully employed to generalize the definition of Heegner points to quadratic extensions that are not necessarily CM. Notably, Guitart, Masdeu and Sengun have defined and numerically computed Stark-Heegner (SH) points in great generality. Their computations strongly support the belief that SH points completely control the Mordell-Weil group of elliptic curves of rank 1.

In this talk I will report on joint works with Gehrmann, Guitart and Masdeu where we propose and numerically compute plectic generalizations of SH points. Inspired by Nekovar and Scholl's conjectures, we expect our points to control Mordell-Weil groups of higher rank elliptic curves.

EYAL GOREN, McGill University
Foliations on Shimura varieties

I report on joint work with Ehud De Shalit (Hebrew University). We consider two kinds of foliations on Shimura varieties. Although our program is quite general, I will focus on two examples to make the topic more digestible in 20 minutes. I will discuss the case of Hilbert modular surfaces and Picard modular surfaces.

JEFFREY HATLEY, Union College
Recent progress in positive rank Iwasawa theory

Many of the earliest results in the Iwasawa theory of elliptic curves and modular forms relied heavily on the finiteness of the relevant Selmer groups. In many natural settings, however, the relevant Selmer groups are not finite. In this talk, we will give a brief survey of some of the recent progress that has been made in generalizing classical results in Iwasawa theory to the positive rank setting.

BORYS KADETS, University of Georgia
Improving Weil bounds for abelian varieties

Weil bounds for an abelian variety A over $\mathbb{F}_q$ give the following estimates $(\sqrt{q}-1)^{2\dim A} \leqslant |A(\mathbb{F}_q)| \leqslant (\sqrt{q}+1)^{2\dim A}$. I will talk about a simple approach to improving these bounds for high-dimensional simple abelian varieties over small fields. For example, when $q = 2,3,4$ the lower Weil bound is vacuous. This method gives $|A(\mathbb{F}_3)| \geqslant 1.359^{\dim A}$ and $|A(\mathbb{F}_4)| \geqslant 2.275^{\dim A}$ for all but finitely many simple abelian varieties A. In contrast, for $q = 2$ an infinite family of simple abelian varieties with only one point is known.

DEBANJANA KUNDU, UBC Vancouver
Arithmetic Statistics and Iwasawa Invariants of Elliptic Curves

In this talk, I will discuss recent results (joint with Anwesh Ray) where we study the average behaviour of the Iwasawa invariants for the Selmer groups of elliptic curves.

MATILDE LALIN, Université de Montréal
The Mahler measure of triangular polynomials

The Mahler measure of a Laurent polynomial P is defined as the integral of $\log|P|$ over the unit torus with respect to the Haar measure. For multivariate polynomials, it often yields special values of L-functions. In this talk we will consider the Mahler measure of polynomials of the form $a(x)+b(x)y+c(x)z \in \mathbb{C}[x,y,z]$ where $a(x), b(x), c(x)$ are products of cyclotomic polynomials. We will exhibit the variety of these formulas, that could range from $\zeta(3)$ and dilogarithms to $L(E,3)$ (the L-function of an elliptic curve). This talk includes joint works with Jarry Gu and Siva Sankar Nair.

ZHENG LIU, University of California, Santa Barbara
p-adic families of Yoshida lifts

We construct a Hida family of Yoshida lifts for two given Hida families of modular forms, and compute the Petersson inner products of its specializations. The key step in the construction is to choose suitable Schwartz functions at p. The computation of the Petersson inner products can be viewed as a generalization of the computation in the works by Bocherer–Dummigan–Schulze-Pillot and Hsieh–Namikawa. Our computation makes use of an equivariant property of the chosen Schwartz functions at p for the action of U_p operators. This is an ongoing joint work with Ming-Lun Hsieh.

ALVARO LOZANO-ROBLEDO, University of Connecticut
This talk is Galois-entangled with Harris Daniels' talk

Let E be an elliptic curve defined over $\mathbb{Q}$. The adelic Galois representation attached to E (this object will be defined during the talk) captures all sorts of interesting information about the arithmetic of the points on $E(\overline{\mathbb{Q}})$, including data about the torsion subgroup, isogenies, and other finer invariants of the curve and its isogeny class. In this talk, and in Harris Daniels' talk, we will give a summary of recent results towards the classification (up to isomorphism) of the possible adelic Galois representations that arise from elliptic curves over $\mathbb{Q}$, and present some recent results of the authors and their collaborators (Garen Chiloyan, Harris Daniels, Jackson Morrow) in this area. Our talks will be mostly self-contained, but very much related... entangled, if you will.

KATHARINA MÜLLER, University of Göttingen
Iwasawa Invariants of fine Slemer groups of congruent abelian varieties

Let K be a number field and let A_1 and A_2 be abelian varieties defined over K. Assume that $A_1[p^l]$ and $A_2[p^l]$ are isomorphic as G_K-modules for some sufficient large l. Let K_∞ be a strongly Σ-admissible p-adic Lie extension (for a suitable set of primes Σ). Generalizing work of Greenberg-Vatsal and Lim-Sujatha we prove an inequality between the μ-invariants of the fine Selmer groups of A_1 and A_2 along the extension K_∞/K. If p^l annihilates the p-primary submodule of both Selmer groups we can even show that the μ-invariants are equal and that the p-primary subgroups are pseudo-isomorphic to each other. If K_∞/K is a $\mathbb{Z}_p$-extension we can derive relations of the corresponding λ-invariants – without assuming that μ vanishes.

This is joint work with Sören Kleine.

GIOVANNI ROSSO, Concordia
Overconvergent Eichler–Shimura morphism for families of Siegel modular forms

Classical results of Eichler and Shimura decompose the cohomology of certain local systems on the modular curve in terms of holomorphic and anti-holomorphic modular forms. A similar result has been proved by Faltings' for the étale cohomology of the modular curve and Falting's result has been partly generalised to Coleman families by Andreatta–Iovita–Stevens. In this talk, based on joint work with Hansheng Diao and Ju-Feng Wu, I will explain how one constructs a morphism from the overconvergent cohomology of GSp_{2g} to the space of families of Siegel modular forms. This can be seen as a first step in an Eichler–Shimura decomposition for overconvergent cohomology and involves a new definition of the sheaf of overconvergent Siegel modular forms using the Hodge–Tate map at infinite level.

R. SUJATHA, University of British Columbia
Refined Iwasawa invariants

This talk is based on joint work with Anwesh Ray. We will introduce Iwasawa theoretic invariants that are refinements of the classical mu-invariant in Iwasawa theory. A conjecture of Greenberg postulates the existence of one member in each isogeny class of elliptic curves over the rational numbers, which has the property that the mu-invariant for its dual Selmer group over the cyclotomic extension vanishes. We will explain how these refined invariants provide a philosophical reasoning for the validity of this conjecture.

JOHN VOIGHT, Dartmouth College
Definite quaternion orders with stable cancellation

Gauss conjectured (in the language of binary quadratic forms) that there are finitely many imaginary quadratic orders of class number 1. There are countless variants of this problem, involving mathematics that is both deep and ongoing. We will survey versions of the class number problem for quaternion orders. In particular, we enumerate all orders with cancellation in the stably free class group. This is joint work with Daniel Smertnig.

JIUYA WANG, Duke University
On Induced Characters with Positivity

Classical result on induced characters shows that for non-cyclic groups any character can be decomposed into rational linear combinations of induced trivial representations from its subgroups. Motivated by recent progress in bounding ℓ-torsion in class groups of number fields, we are led to ask a similar question, but with the extra positivity constraints for the rational coefficients. We will give a full answer towards this question for regular representations, and we will also introduce its application in studying class groups. This is a joint work of Cui, Fleischer, Gu and Liu.