Arithmetic Geometry
Géométrie arithmétique
(Org: **Eyal Goren** (McGill) and/et **Steve Kudla** (Toronto))

PATRICK ALLEN, McGill University
Modularity of some PGL(2,5) representations

Serre's conjecture, proved by Khare and Wintenberger, states that every odd two dimensional mod p representation of the absolute Galois group of the rationals comes from a modular form. This admits a natural generalization to totally real fields, but even the real quadratic case seems completely out of reach. I'll discuss some of the difficulties one encounters and then discuss some new cases that can be proved when p = 5. This is joint work with Chandrashekhar Khare and Jack Thorne.

LEA BENEISH, McGill University
Fields generated by points on superelliptic curves

We give an asymptotic lower bound on the number of field extensions generated by algebraic points on superelliptic curves over $\mathbb{Q}$ with fixed degree n, discriminant bounded by X, and Galois closure S_n. For C a fixed curve given by an affine equation $y^m = f(x)$ where $m \geq 2$ and deg $f(x) = d \geq m$, we find that for all degrees n divisible by gcd(m, d) and sufficiently large, the number of such fields is asymptotically bounded below by X^{c_n}, where $c_n \to 1/m^2$ as $n \to \infty$. This bound is determined explicitly by parameterizing x and y by rational functions, counting specializations, and accounting for multiplicity. We then give geometric heuristics suggesting that for n not divisible by gcd(m, d), degree n points may be less abundant than those for which n is divisible by gcd(m, d). Namely, we discuss the obvious geometric sources from which we expect to find points on C and discuss the relationship between these sources and our parametrization. When one a priori has a point on C of degree not divisible by gcd(m, d), we argue that a similar counting argument applies. This talk is based on joint work with Christopher Keyes.

LUCA CANDELORI, Wayne State University
Topological Hecke Operators

Topological Hecke operators were first defined by Baker in the 1980s as stable operations on elliptic homology, coinciding with the classical level one Hecke operators when evaluated on the one-point space. Very few calculations have been done with them since. In this talk, we provide the foundations for a study of eigenforms for the action of topological Hecke operators acting on the holomorphic elliptic homology of various topological spaces. We prove a multiplicity one theorem for some classes of topological spaces, and we give examples of finite CW-complexes for which multiplicity one fails. We also develop some abstract "derived eigentheory" whose motivating examples arise from the failure of classical Hecke operators to commute with multiplication by various Eisenstein series. Part of this "derived eigentheory" is an identification of certain derived Hecke eigenforms as the obstructions to extending topological Hecke eigenforms from the top cell of a CW-complex to the rest of the CW-complex. Using these obstruction classes together with our multiplicity one theorem, we calculate the topological Hecke eigenforms explicitly, in terms of pairs of classical modular forms, on all 2-cell CW complexes obtained by coning off an element in $\pi_n(S^m)$ which stably has Adams-Novikov filtration 1. These explicit examples provide a surprising connection between torsion in the stable homotopy groups of spheres and congruences between the coefficients of level one modular forms.

FRANCESC CASTELLA, University of California, Santa Barbara
Iwasawa theory for $\mathrm{GL}_2 \times \mathrm{GL}_2$ and diagonal cycles

In this talk I will explain the construction, in joint work with Raul Alonso Rodriguez and Oscar Rivero, of an anticyclotomic Euler system for the tensor product of the Galois representations attached to two modular forms arising from generalized

Gross–Kudla–Schoen diagonal cycles and their variation in p-adic families. As applications of this construction, we prove new cases of the Bloch–Kato conjecture in analytic rank zero and a divisibility towards an Iwasawa main conjecture.

SAMIT DASGUPTA, Duke University
On the Brumer-Stark Conjecture and Refinements

We will present recent results on the Brumer-Stark conjecture and refinements, with applications to explicit class field theory for totally real fields. This is joint work with Mahesh Kakde, Jesse Silliman, and Jiuya Wang.

LENNART GEHRMANN, Universität Duisburg-Essen
On quaternionic rigid meromorphic cocyles

Recently, Darmon and Vonk initiated the theory of rigid meromorphic cocycles for the group $\mathrm{SL}_2(\mathbb{Z}[1/p])$. One of their first major results is the algebraicity of the divisor associated to such a cocycle. Their proof does not easily generalize to more general situations as it relies on rather explicit methods. In particular, it involves computations with generators of the group $\mathrm{SL}_2(\mathbb{Z})$.

I will explain an alternative proof of their result that only uses standard homological properties of arithmetic groups, e.g. Bieri-Eckmann duality. An advantage of this approach is that it also works for $\mathfrak{p}$-arithmetic subgroups of inner forms of SL_2 over arbitrary number fields.

MATHILDE GERBELLI-GAUTHIER, Institute for Advanced Study
Growth of Cohomology of Arithmetic Groups and Endoscopy

I will discuss growth of Betti numbers in towers of Shimura varieties associated to unitary groups. Specifically, I will outline a strategy to bound the growth for low degrees of cohomology using automorphic representations and the phenomenon of endoscopy.

QIRUI LI, University of Toronto
Linear Arithmetic Fundamental Lemma and Intersection numbers for CM cycles on Lubin—Tate spaces

The Guo-Jacquet Fundamental Lemma is a higher dimensional generalization of the local field analogue of the Waldspurger formula. It has an arithmetic generalization called the Linear Arithmetic Fundamental Lemma. It is conjectured by Wei Zhang interpreting the derivative of certain orbital integral into certain intersection number of Lubin-Tate spaces, which is a local analogue of the Gross-Zagier formula. We will introduce the known results for the linear Arithmetic Fundamental Lemma, and the intersection number formula for Lubin—Tate spaces. After a joint work with Ben Howard, we also discovered a bi-quadratic generalization of the conjecture.

MICHAEL LIPNOWSKI, McGill University
Story about a hyperbolic 3-manifold

The Seifert-Weber dodecahedral space is a famous closed hyperbolic 3-manifold, one of the first to be discovered. I'll discuss how some computations thereon, undertaken together with Francesco Lin, inspired a conjecture about unlikely intersections of geodesics on hyperbolic manifolds.

ZHENG LIU, University of California, Santa Barbara
The doubling archimedean zeta integrals for unitary groups

In order to construct p-adic L-functions for symplectic and unitary groups by using the doubling method and verify the interpolation properties predicted by the conjecture of Coates–Perrin-Riou, special archimedean test sections need to be chosen and a doubling archimedean zeta integral needs to be calculated for holomorphic discrete series. When the holomorphic discrete series is of scalar weight, it has been done by Bocherer-Schmidt and Shimura. In this talk, I will discuss computing the archimedean zeta integrals for unitary groups when the holomorphic discrete series is of general weight. This is a joint work with Ellen Eischen.

MATTEO LONGO, Università di Padova
On the Equivariant Tamagawa Number conjecture for modular forms

The Equivariant Tamagawa Number Conjecture was formulated by Bloch and Kato in 1990, and can be seen as a generalisation to motives of the Birch and Swinnerton-Dyer Conjecture for elliptic curves. In the latter case, the validity of the p-part of the Birch and Swinnerton-Dyer Conjecture for ordinary primes p is known when the analytic rank of the rational elliptic curve $E/\mathbb{Q}$ is equal to 1. We prove a similar result for the p-part of the Bloch-Kato conjecture for motives attached to newforms. For this, we prove a version of Kolyvagin's Conjecture for modular forms, from which we deduce the p-part of the Tamagawa Number Conjecture. This is a work in collaboration with Stefano Vigni.

ALICE POZZI, Imperial College London
Derivatives of Hida families and rigid meromorphic cocycles

A rigid meromorphic cocycle is a class in the first cohomology of the group $\mathrm{SL}_2(\mathbb{Z}[1/p])$ acting on the non-zero rigid meromorphic functions on the Drinfeld p-adic upper half plane by Möbius transformation. Rigid meromorphic cocycles can be evaluated at points of real multiplication, and their values conjecturally lie in the ring class field of real quadratic fields, suggesting striking analogies with the classical theory of complex multiplication.

In this talk, we discuss the relation between the derivatives of certain p-adic families of Hilbert modular forms and rigid meromorphic cocycles. We explain how the study of congruences between cuspidal and Eisenstein families allows us to show the algebraicity of the values of a certain rigid meromorphic cocycle at real multiplication points.

This is joint work with Henri Darmon and Jan Vonk.

KARTIK PRASANNA, University of Michigan
Quaternionic modular forms, cycles and L-functions

I will give an overview of some questions about algebraic cycles and L-functions in the context of quaternionic modular forms, describe some recent progress (based on joint work with Ichino) and outline some problems that still remain.

GIOVANNI ROSSO, Concordia
Specialness for non-archimedean varieties

Since the fundamental work of Faltings on Mordell's conjecture, many conjectures have been made concerning the problems of when rational points of a variety over a number field are (potentially) Zariski dense. Varieties whose rational points are (potentially) Zariski dense are called special, and Campana characterised these varieties as the ones that (loosely speaking) don't admit fibrations to varieties of general type. Conjecturally, this is equivalent to the fact that complex analytification of the variety is Brody-special; that is, it admits a dense entire curve. Inspired by the notion of Brody-special, in a joint work with Jackson Morrow, we introduced the notion of K-analytically special varieties over an algebraically closed non archimedean field K. In this presentation, I shall explain this definition and prove several results (K-analytically special sub-varieties of semi-abelian varieties are translate of semi-abelian varieties; K-analytically special varieties don't dominate pseudo-K-analytically Brody hyperbolic variety) that support the fact that our notion is the right one to test specialness in p-adic analytic geometry.

SIDDARTH SANKARAN, University of Manitoba
Arithmetic special cycles and Jacobi forms

Kudla's conjectural program predicts relations between certain (arithmetic) "special" cycles on Shimura varieties and the Fourier coefficients of automorphic forms. In this talk, I'll focus on the case of a compact orthogonal Shimura variety X; by augmenting the special cycles with appropriate choices of Green currents, we obtain classes in the arithmetic Chow group of X (viewed as a variety over its reflex field). After reviewing these notions, I'll describe a modularity result identifying generating series built from these special cycles as Jacobi forms, yielding evidence for Kudla's conjectures in this setting.

ILA VARMA, Unviersity of Toronto
Malle's Conjecture for octic D_4-fields

We consider the family of normal octic fields with Galois group D_4, ordered by their discriminant. In forthcoming joint work with Arul Shankar, we verify the strong form of Malle's conjecture for this family of number fields, obtaining the order of growth as well as the constant of proportionality. In this talk, we will discuss and review the combination of techniques from analytic number theory and geometry-of-numbers methods used to prove this and related results.

JOHN VOIGHT, Dartmouth College
Sato-Tate groups and modularity for atypical abelian surfaces

We discuss in detail what it means for an abelian surface A over a number field to be modular, organizing conjectures and theorems that associate to A a modular form with matching L-function. The explicit description of this modular form depends on the real Galois endomorphism type of A, or equivalently on its Sato–Tate group. For A defined over the rational numbers, this description can involve classical, Bianchi, or Hilbert modular forms; and for each possibility, we provide a genus 2 curve with small conductor from which it arises. This is joint work with Andrew Booker, Jeroen Sijsling, Andrew Sutherland, and Dan Yasaki.

JAN VONK, University of Leiden
Modular generating series of RM invariants

In this short talk, I will discuss some recent progress on the modularity of generating series of RM invariants that arise as special values of rigid cocycles. This provides results in the spirit of the Kudla programme, whose intrinsically p-adic nature furnishes a direct connection with class field theory. This work is joint with Henri Darmon and Alice Pozzi.

TONGHAI YANG, UW-Madison
Kudla-Rapoport conjecture at a ramified prime

This is a joint work with Qiao He and Yousheng Shi. One important part of the Kudla program is the so-called Arithmetic Siegel-Weil formula, which reveals some deep relation between the Fourier coefficients of some incoherent Eisenstein series and arithmetic Heigh pairing on a Shimura variety (of unitary type (n,1) or orthogonal (n, 2)). To prove it for non-singular coefficients, it amounts to prove a local identiy—the so-called Kudla-Rapoport conjecture or local arithmetic SIegel-Weil formula— and a global counting identity (Siegel-Weil formula). Chao Li and Wei Zhang found a beautiful proof of the Kudla-Rapoport conjecture at unramified primes. In this talk, we will discuss its analogue at ramified primes when n=1, where some modification is needed. If time permits, we might describe possible generalization of this work for general n.