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Topological Hecke Operators

Topological Hecke operators were first defined by Baker in the 1980s as stable operations on elliptic homology, coinciding with the classical level one Hecke operators when evaluated on the one-point space. Very few calculations have been done with them since. In this talk, we provide the foundations for a study of eigenforms for the action of topological Hecke operators acting on the holomorphic elliptic homology of various topological spaces. We prove a multiplicity one theorem for some classes of topological spaces, and we give examples of finite CW-complexes for which multiplicity one fails. We also develop some abstract "derived eigentheory" whose motivating examples arise from the failure of classical Hecke operators to commute with multiplication by various Eisenstein series. Part of this "derived eigentheory" is an identification of certain derived Hecke eigenforms as the obstructions to extending topological Hecke eigenforms from the top cell of a CW-complex to the rest of the CW-complex. Using these obstruction classes together with our multiplicity one theorem, we calculate the topological Hecke eigenforms explicitly, in terms of pairs of classical modular forms, on all 2-cell CW complexes obtained by coning off an element in $\pi_n(S^m)$ which stably has Adams-Novikov filtration 1. These explicit examples provide a surprising connection between torsion in the stable homotopy groups of spheres and congruences between the coefficients of level one modular forms.