LEA BENEISH, McGill University

*Fields generated by points on superelliptic curves*

We give an asymptotic lower bound on the number of field extensions generated by algebraic points on superelliptic curves over $\mathbb{Q}$ with fixed degree $n$, discriminant bounded by $X$, and Galois closure $S_n$. For $C$ a fixed curve given by an affine equation $y^m = f(x)$ where $m \geq 2$ and $\deg f(x) = d \geq m$, we find that for all degrees $n$ divisible by $\gcd(m, d)$ and sufficiently large, the number of such fields is asymptotically bounded below by $X^{c_n}$, where $c_n \to 1/m^2$ as $n \to \infty$. This bound is determined explicitly by parameterizing $x$ and $y$ by rational functions, counting specializations, and accounting for multiplicity. We then give geometric heuristics suggesting that for $n$ not divisible by $\gcd(m, d)$, degree $n$ points may be less abundant than those for which $n$ is divisible by $\gcd(m, d)$. Namely, we discuss the obvious geometric sources from which we expect to find points on $C$ and discuss the relationship between these sources and our parametrization. When one a priori has a point on $C$ of degree not divisible by $\gcd(m, d)$, we argue that a similar counting argument applies. This talk is based on joint work with Christopher Keyes.