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Attaining the exponent 5/4 for the sum product problem in finite fields

The sum-product problem is to show, for any finite set A, that one of the sum set $A + A$ or product set AA must be large in cardinality. Progress on this problem over finite fields lags behind its counterpart in the reals, where notably in 1997 Elekes used the Szemerédi-Trotter theorem to obtain the exponent $5/4$; this exponent has since advanced in the reals. In a joint work with Ali Mohammadi, we show that if $A \subseteq \mathbb{F}_p$ has cardinality $|A| \ll p^{1/2}$ then we match Elekes' bound. That is, we show that

$$\max\{|A \pm A|, |AA|\} \lesssim |A|^{\frac{5}{4}}.$$

This improves the exponent of $11/9$ by Rudnev, Shakan and Shkredov from 2018.