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Weissler's conjecture on the Hamming cube

Let $1 \leq p \leq q < \infty$, and $z \in \mathbb{C}$. We show that the Hermite operator $\exp(z\Delta)$ is bounded from $L_p(\{-1, 1\}^n)$ to $L_q(\{-1, 1\}^n)$ with norm independent of n if and only if $|p - 2 - e^{2z}(q - 2)| \leq p - |e^{2z}|q$. This solves an old open problem in complex hypercontractivity theory on the Hamming. Certain cases of the triples (p, q, z) were characterized by Bonami (1970); Beckner (1975); and Weissler (1979). Several applications will be presented. Work in progress with Fedja Nazarov.