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*Solution to several problems regarding tensor products and crossed products of $C^*$- and von Neumann algebras*

We present solutions to several problems concerning crossed products and tensor products of operator algebras. The common theme is our use of completely bounded module maps.

We prove that a locally compact group $G$ has the approximation property (AP) if and only if a non-commutative Fejér theorem holds for the associated $C^*$- or von Neumann crossed products. We deduce that the AP always implies exactness. This generalizes a result of Haagerup–Kraus, and answers a question by Li. We also answer a problem of Bédos–Conti on discrete $C^*$-dynamical systems, and a question by Anoussis–Katavolos–Todorov on bimodules over the group von Neumann algebra $VN(G)$ for all locally compact groups $G$ with the AP. In our approach, we develop a notion of Fubini crossed product for locally compact groups, and a dynamical version of the AP for actions. (Joint work with J. Crann.)

It has been open for almost 40 years to characterize when the projective Banach tensor square $\mathcal{A} \otimes_\gamma \mathcal{A}$ of a $C^*$-algebra $\mathcal{A}$ is Arens regular. We solve this problem for arbitrary $C^*$-algebras: Arens regularity is equivalent to $\mathcal{A}$ having the Phillips property; hence, it is encoded in the geometry of $\mathcal{A}$. For a von Neumann algebra $\mathcal{A}$, we conclude that $\mathcal{A} \otimes_\gamma \mathcal{A}$ is Arens regular only if $\mathcal{A}$ is finite-dimensional. We also show that this does not generalize to non-selfadjoint operator algebras. For commutative $C^*$-algebras $\mathcal{A}$, we prove that the centre of $(\mathcal{A} \otimes_\gamma \mathcal{A})^{**}$ is Banach algebra isomorphic to the extended Haagerup tensor product $\mathcal{A}^{**} \otimes_{eh} \mathcal{A}^{**}$.