Topology
Topologie

(Org: **Martin Frankland**, **Donald Stanley** and/et **Paul Arnaud Songhafouo Tsopméné** (Regina))

ANTHONY BAHRI, Rider University

Polyhedral products and their applications

A polyhedral product is a natural subspace of a Cartesian product specified by a simplicial complex. Though they arose from the topological approach to toric geometry, their utility has expanded rapidly in recent years into areas which include: representation theory, combinatorics, geometric group theory, the topology of toric spaces, free groups and monodromy, complements of subspace arrangements, number theory, graph products, quadratic algebras, arachnid mechanisms and homotopy theory. In this talk I shall describe a new approach to their cohomology and discuss a few applications.

JEFFREY CARLSON, University of Toronto

Local integration in equivariant cobordism theory

It has long been known that the equivariant complex T-cobordism class $[M]$ of an stably complex manifold M equipped with the action of a torus T is uniquely determined by the equivariant normal bundle $\nu(M^T)$ to its fixed point set, and tom Dieck and Lü–Wang respectively showed $[M]$ is also determined by its equivariant-K-theoretic and Borel-cohomological Chern numbers.

Each of these results constructs an injection of the bordism ring $\Omega_*^{U,T}$ into local data, but none identifies the image. Not all local data is possible, by the Atiyah–Bott/Berline–Vergne ("ABBV") localization theorem, which expresses equivariant characteristic numbers in terms of tangent and normal data on M^T, thus imposing identities involving mixed Chern numbers and tangent and normal representation data. It is natural to wonder whether these ABBV identities cut out the image of $\Omega_*^{U,T}$ or there are further constraints.

For isolated fixed points, normal data essentially comprises a list of T-representations. In the case of GKM torus actions and semifree circle actions, we show, in some cases via concrete construction, that there are no other constraints: every list of representation data consistent with the ABBV identities in fact occurs.

This work builds on that of Alastair Darby in the GKM case and is joint with Adina Elisheva Gamse and Yael Karshon.

TYRONE GHASWALA, University of Manitoba

Promoting circle actions to actions on the real line

Circularly-orderable and left-orderable groups play an important, and sometimes surprising, role in low-dimensional topology and geometry. For example, these combinatorial conditions completely characterize when a countable group acts on a 1-manifold. Through the so-called L-space conjecture, left-orderability of the fundamental group of rational homology 3-sphere is connected to analytic and topological properties of the manifold. I will present new necessary and sufficient conditions for a circularly-orderable group to be left-orderable, and introduce the obstruction spectrum of a circularly-orderable group. This raises a plethora of intriguing questions.

This is joint work with Jason Bell and Adam Clay.

RACHEL HARDEMAN, University of Calgary

An Introduction to A-Homotopy Theory: A Discrete Homotopy Theory for Graphs

A-homotopy theory was invented by R. Aktin in the 1970s and further developed by H. Barcelo and others in the early 2000s as a combinatorial version of homotopy theory. This theory respects the structure of a graph, distinguishing between vertices and edges. While in classical homotopy theory all cycles are equivalent to the circle, in Ahomotopy theory the 3-cycle and 4-cycle are contractible and all larger cycles are equivalent to the circle.

In this talk, we will examine the fundamental group in A-homotopy from the perspective of covering spaces. We will also establish explicit lifting criteria and examine the role of the 3-cycles and 4-cycles in these criteria.

ROBIN KOYTCHEFF, University of Louisiana at Lafayette
The Taylor tower for the space of knots and finite-type knot invariants

I will discuss the Taylor tower for the space of long 1-dimensional knots in Euclidean space, which comes from Goodwillie-Weiss functor calculus. When the codimension is at least 3, this tower converges to the space of knots, while in the classical case of codimension 2, all real-valued finite-type invariants factor through it. With Budney, Conant, and Sinha, we constructed a homotopy-commutative multiplication on each stage of the tower compatible with stacking long knots via the evaluation map. This helped us provide evidence for a conjecture that all abelian-group-valued finite-type invariants factor through the tower. Topics of ongoing and planned joint work include actions of the cactus operad and splicing operad on the tower, as well as computations in the homotopy spectral sequence of the tower.

ROBIN KOYTCHEFF, University of Louisiana at Lafayette
Operadic decompositions of spaces of string links

Budney showed that the space of long knots in 3-space is the free 2-cubes object on the space of prime knots, generalizing prime decomposition of knots from isotopy classes to the space of knots. He also showed that the space of long knots is freely generated over a splicing operad by the subspace of torus and hyperbolic knots, generalizing satellite decomposition of knots from isotopy classes to the space level. In joint work with Burke, we constructed a colored operad for string link infection, generalizing Budney's splicing operad from knots to links. We then decomposed a subspace of 2-component string links over a certain suboperad of the infection operad. We conjecture that the full space of 2-string links admits a decomposition involving a suboperad related to the Swiss cheese operad. This conjecture is the subject of planned work with Songhafouo Tsopméné.

PATRICK NAYLOR, University of Waterloo
Trisections and twists of 4-manifolds

Trisections were introduced by Gay and Kirby in 2013 as a way to study 4-manifolds. They are very similar in spirit to Heegaard splittings of 3-manifolds, and have the advantage of changing problems about manifolds into problems about diagrams. In this talk, I will give a brief introduction to trisections, and explain how they can be used to reprove a theorem of Katanaga, Saeki, Teragaito, and Yamada that relates Gluck and Price twists of 4-manifolds. This answers a recent question of Seungwon Kim and Maggie Miller.

CIHAN OKAY, University of British Columbia
Mod-ℓ homotopy type of the classifying space for commutativity

The classifying space for commutativity, denoted by $B_{\mathsf{com}}G$, of a Lie group G is assembled from commuting tuples in G as a subspace of the usual classifying space BG. The resulting space classifies principal G-bundles whose transition functions generate an abelian subgroup of G whenever they are simultaneously defined. The relationship between the homotopy type of G and the space $B_{\mathsf{com}}G$ is much more interesting, and non-trivial compared to the case of BG. In this talk, I will present a work, joint with Ben Williams, where we study the mod-ℓ homotopy type of $B_{\mathsf{com}}G$ at a prime ℓ. The techniques involve a homotopy colimit decomposition over a topological category generalizing the construction of Adem-Gomez and application of results on mapping spaces between classifying spaces of compact Lie groups due to Dwyer-Wilkerson. We show that for a connected compact Lie group the mod-ℓ homotopy type of $B_{\mathsf{com}}G$ depends on the mod-ℓ homotopy type of BG.

KATE POIRIER, City University of New York - New York City College of Technology
Directed planar trees, V-infinity algebras, and string topology

Stasheff's associahedra are polyhedra that provide a model for algebras whose products are associative up to homotopy. In this talk, we introduce "assocoipahedra," which model $\mathcal{V}_\infty$ algebras—algebras with a product and a co-inner product and relations that hold up to homotopy. Where associahedra are described combinatorially in terms of rooted planar trees, assocoipahedra are generalizations described in terms of directed planar trees. We use the structure of assocoipahedra to describe Tradler–Zeinalian's algebraic string operations on the Hochschild complex of a $\mathcal{V}_\infty$ algebra and Drummond-Cole–Poirier–Rounds's corresponding string topology operations on the singular chains of the free loop space of a manifold. We also use this structure to show that the operad governing $\mathcal{V}_\infty$ algebras is Koszul. This is joint work with Thomas Tradler.

STEVEN RAYAN, University of Saskatchewan
The quiver at the bottom of the twisted nilpotent cone on $\mathbb{P}^1$

For the moduli space of Higgs bundles on a Riemann surface of positive genus, critical points of the natural Morse-Bott function lie along the nilpotent cone of the Hitchin fibration and are representations of A-type quivers in a twisted category of holomorphic bundles. The fixed points that globally minimize the function are representations of A_1. For twisted Higgs bundles on the projective line, the quiver describing the bottom of the cone is more complicated. We determine it and show that the moduli space is topologically connected whenever the rank and degree are coprime.

KRISHANU SANKAR, University of British Columbia
Mod 2 cohomology and the braid group

A classical theorem of Mahowald states that the Eilenberg-Maclane spectrum of Z/2 is the Thom spectrum of the stable Hopf bundle on the double loopspace of the 3-sphere. Thus, HZ/2 is filtered by Thom spectra on the classifying spaces of the braid groups. There is a closely related filtration built from symmetric powers, and these two filtrations capture the intricate behavior of mod 2 power operations.

I'll discuss recent work which generalizes this story to the equivariant setting, including two actions of the cyclic group of order two on the braid group and a geometric interpretation of equivariant mod 2 homology in terms of bundles.

LAURA SCULL, Fort Lewis College
Transitive Groupoids with Interesting Topological Properties

I will discuss some examples of interesting transitive topological groupoids from the literature. This talk is an offshoot of a project with J. Watts and C. Farsi, where we studied the actions of transitive groupoids. Under normal topological conditions, we can reduce the action of a transitive groupoid to the action of a single isotropy group on a fibre. However, we ran across examples of some more unusual topological groupoids where this is not possible. I will discuss examples from Muhly-Renault-Williams and Buneci.

BEN WILLIAMS, University of British Columbia
$\mathbb{A}^1$*-homotopy and a conjecture of Suslin*

This is joint work with Aravind Asok and Jean Fasel.

Fix an infinite perfect field k.

The $\mathbb{A}^1$-homotopy theory of SL_n is intimately related with algebraic K-theory. Specifically, for $i \in \{1, \ldots, n-2\}$, one has $\pi_i^{\mathbb{A}^1}(SL_n)(k) = K_i(k)$. The boundary case of $i = n-1$ has something to do with the interface between rank-n vector bundles and K-theory, which has been extensively studied. We give a partial calculation of $\pi_2^{\mathbb{A}^1}(SL_3)$ and a complete calculation of

$\pi_3^{\mathbb{A}^1}(SL_4)$, and we show how this (partly in the case of $n = 3$) answer a question of Suslin's from 1984 regarding the homology of the group $SL_n(k)$.