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\mathbb{A}^1 -homotopy and a conjecture of Suslin

This is joint work with Aravind Asok and Jean Fasel.

Fix an infinite perfect field k .

The \mathbb{A}^1 -homotopy theory of SL_n is intimately related with algebraic K -theory. Specifically, for $i \in \{1, \dots, n-2\}$, one has $\pi_i^{\mathbb{A}^1}(SL_n)(k) = K_i(k)$. The boundary case of $i = n-1$ has something to do with the interface between rank- n vector bundles and K -theory, which has been extensively studied. We give a partial calculation of $\pi_2^{\mathbb{A}^1}(SL_3)$ and a complete calculation of $\pi_3^{\mathbb{A}^1}(SL_4)$, and we show how this (partly in the case of $n = 3$) answer a question of Suslin's from 1984 regarding the homology of the group $SL_n(k)$.