Probabilistic Methods in Geometric Functional Analysis and Convexity
Méthodes probabilistes en analyse et en convexité fonctionnelle géométrique
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SUSANNA DANN, Universidad de los Andes
Affine isoperimetric inequalities on flag manifolds.

We introduce r-flag affine quermassintegrals and their dual versions. These quantities generalize the affine and dual affine quermassintegrals as averages on flag manifolds (where the Grassmannian can be considered as a special case). We establish affine and linear invariance properties and sharp inequalities extending all known results to this new setting. We also discuss inequalities for functional forms of these new quantities. This is joint work with Grigoris Paouris and Peter Pivovarov.

VICTOR GLASGO, Case Western Reserve University
Gravitational illumination bodies (Preliminary report)

We introduce a new class of convex bodies, the gravitational illumination bodies. We show some of their properties and explore their relation to affine surface area of convex bodies.

Based on joint work with Andreas Kreuml and Elisabeth M. Werner.

QINGZHONG HUANG, Department of Mathematics and Statistics, Memorial University of Newfoundland
The L_p John ellipsoid for Sobolev functions

The L_p John ellipsoid for convex bodies was introduced by Lutwak, Yang and Zhang, which contains the John ellipsoid ($p = \infty$), the LYZ ellipsoid ($p = 2$), and the Petty ellipsoid ($p = 1$) as special cases. In this talk, we will discuss the L_p John ellipsoids for Sobolev functions and for log-concave functions. Moreover, a functional Blaschke-Santal$\acute{o}$ inequality for L_2 John ellipsoid will be presented. This talk is based on the joint work with Ai-jun Li and Deping Ye.

PAATA IVANISVILI, University of California, Irvine
Weissler's conjecture on the Hamming cube

Let $1 \leq p \leq q < \infty$, and $z \in \mathbb{C}$. We show that the Hermite operator $\exp(z\Delta)$ is bounded from $L_p(\{-1,1\}^n)$ to $L_q(\{-1,1\}^n)$ with norm independent of n if and only if $|p-2-e^{2z}(q-2)| \leq p - |e^{2z}|q$. This solves an old open problem in complex hypercontractivity theory on the Hamming. Certain cases of the triples (p,q,z) were characterized by Bonami (1970); Beckner (1975); and Weissler (1979). Several applications will be presented. Work in progress with Fedja Nazarov.

ALEXANDER LITVAK, University of Alberta
On the volume ratio between convex bodies

In this talk I'll survey known results on volume ratio between convex bodies. Cubical and simplex ratios will be discussed as well as the general case and Banach-Mazur type distances.

GALYNA LIVSHYTS, Georgia Institute of Technology
Smallest singular value of inhomogeneous random square matrices via double counting and random rounding

We show that the small ball behavior of random square matrices is optimal under minimal assumptions. An important step in the proof is showing that the "random normal" — random vector orthogonal to a collection of (n-1) random vectors — has very good behavior, and its projection onto another random vector is not concentrated on any short interval. Previously, such result was known only under additional i.i.d. assumption, and the key technique leading to it was developed by Rudelson and Vershynin. Their approach, however, does not work without the i.i.d. requirement.

In order to show that the random normal is "good" we prove that the set of "bad" vectors is small: we construct a net on it of small cardinality. This net is a subset of a net on the sphere with simple lattice structure, and its construction relies on the method of random rounding. To show that the cardinality of this subset is small, we show that most of the vectors on a lattice are "good", and therefore cannot be close to "bad" vectors. This key step is done via harmonic-analytic techniques from discrepancy theory. This is a joint work with K. Tikhomirov and R. Vershynin.

MOKSHAY MADIMAN, University of Delaware
Sharp moment-entropy inequalities for log-concave distributions

We show that the uniform distribution minimises entropy among all symmetric log-concave distributions with fixed variance, and also discuss some related ideas. (Based on joint work with Piotr Nayar and Tomasz Tkocz.)

ARNAUD MARSIGLIETTI, University of Florida
Hyperplane conjecture and central limit theorem

The hyperplane conjecture, raised by Bourgain in 1986, is a major unsolved problem in high-dimensional geometry. We discuss a probabilistic approach toward solving the hyperplane conjecture, which consists of rewriting the problem as a (strong form of the) central limit theorem.

SERGII MYROSHNYCHENKO, University of Alberta

PIOTR NAYAR, University of Warsaw
The log-concave moment problem

Let us fix real numbers $p_1, \ldots, p_n > -1$. We say that a finite sequence of real numbers $m_1, \ldots, m_n$ is admissible, if there exists a symmetric log-concave function $f : \mathbb{R} \to \mathbb{R}$ such that $m_i = \int |t|^{p_i} f(t)\mathrm{d}t$ for all $i = 1, \ldots, n$. During the talk I will provide a description of all admissible sequences. Based on a joint work with A. Eskenazis and T. Tkocz.

YAIR SHENFELD, Princeton
Extremals in Minkowski's quadratic inequality

The ball uniquely minimizes the surface area among all convex bodies with fixed volume. On the other hand, if one wishes to control also the mean-width of the bodies, for example, then there are many minimizers whose shapes are quite strange. The characterization of such bodies follows from understanding the equality cases in Minkowski's quadratic inequality. This problem was open for more than hundred years. In this talk I will discuss the problem and its solution. (Joint work with Ramon van Handel.)

BOAZ SLOMKA, Weizmann Institute of Science
On Hadwiger's covering problem

A long-standing open problem, known as Hadwiger's covering problem, asks what is the smallest natural number $N(n)$ such that every convex body in $\mathbb{R}^n$ can be covered by a union of the interiors of at most $N(n)$ of its translates.

In this talk, I will present a recent work in which we prove a new upper bound for $N(n)$. This bound improves Rogers' previous best bound, which is of the order of $\binom{2n}{n} n \ln n$, by a sub-exponential factor. Our approach combines ideas from previous work with tools from asymptotic geometric analysis. As a key step, we use thin-shell estimates for isotropic log-concave measures to prove a new lower bound for the maximum volume of the intersection of a convex body K with a translate of $-K$. We further show that the same bound holds for the volume of $K \cap (-K)$ if the center of mass of K is at the origin.

If time permits we shall discuss some other methods and results concerning this problem and its relatives.

Joint work with H. Huang, B. Vritsiou, and T. Tkocz

KATERYNA TATARKO, University of Alberta
On the solution to the reverse isoperimetric problem

The classical isoperimetric problem asks which domain, among all domains with a

fixed surface area, has maximal volume. The question has a long and beautiful history and has been generalized to a variety of different settings. On the other hand, one can formulate the reverse isoperimetric problem: under which conditions can one minimize the volume among all domains of a given constraint.

In this talk we consider a class of λ-concave bodies in $\mathbb{R}^{n+1}$; that is, convex bodies with the property that each of their boundary points supports a tangent ball of radius $1/\lambda$ that lies locally (around the boundary point) inside the body. In this class, we solve a reverse isoperimetric problem: we show that the convex hull of two balls of radius $1/\lambda$ (a sausage body) is a unique volume minimizer among all λ-concave bodies of given surface area. This is joint work with Roman Chernov and Kostiantyn Drach.

KONSTANTIN TIKHOMIROV, Georgia Institute of Technology
Small ball probability for the condition number of random matrices

Let A be an n by n random matrix with i.i.d. entries of zero mean, unit variance and a bounded subgaussian moment. We show that the smallest singular value of A, rescaled by the square root of n, is a subgaussian random variable. Although the statement can be obtained as a combination of known results and techniques, it was not noticed in the literature before. As a key step of the proof, we apply estimates for the intermediate singular values of A obtained (under some additional assumptions) by Hoi Nguyen. The talk is based on a joint work with Alexander Litvak and Nicole Tomczak-Jaegermann.

ELISABETH WERNER,
Entropy inequalities for log concave functions

We discuss new reverse log Sobolev type inequalities for log concave functions that strengthen existing ones. Equality characterizations in these inequalities lead to a Monge Ampere differential equation. We investigate the solutions of this Monge Ampere equation.

Based on joint work with U. Caglar and A. V. Kolesnikov.

JIE XIAO,
Gaussian BV Capacity

Since the Gaussian perimeter exists as an (n-1)-dimensional area with the standard Gaussian density, the Gaussian space merits a geometrical capacitary analysis on the bounded variation functions which are differentiable in the weakest measure theoretic sense. This talk will address a Gaussian analogy of the bounded variation capacity of a subset of Euclidean space, which is treated as a foundation of L.Liu-J.Xiao-D.Yang-W.Yuan's monograph: Gaussian Capacity Analysis, Lecture Notes in Mathematics 2225, Springer.

SUDAN XING, Memorial University of Newfoundland
The general dual-polar Orlicz-Minkowski problem

In this talk, I will discuss the general dual-polar Orlicz-Minkowski problem, which is "polar" to the recently initiated general dual Orlicz-Minkowski problem and "dual" to the newly proposed polar Orlicz-Minkowski problem. The problem states as follows:

Under what conditions on a nonzero finite Borel measure μ defined on the unit sphere, continuous functions $\varphi : (0,\infty) \to (0,\infty)$ and $G :\ (0,\infty)\times S^{n-1} \to (0,\infty)$ can we find a convex body K (with the origin in its interior) solving the following optimization problems

$$\inf / \sup \left\{ \int_{S^{n-1}} \varphi(h_Q(u))d\mu(u) : Q \in \widetilde{\mathcal{B}} \right\},$$

where $\widetilde{\mathcal{B}} = \left\{Q \in \mathcal{K}^n_{(o)} :\ \widetilde{V}_G(Q^\circ) = \widetilde{V}_G(B^n)\right\}$ with B^n the unit ball and $\widetilde{V}_G$ the general dual volume. In particular, we will present the existence, continuity and uniqueness of the solutions for the general dual-polar Orlicz-Minkowski problem. This talk is based on a joint work with Deping Ye and Baocheng Zhu.

DEPING YE, Memorial University of Newfoundland
The polar Orlicz-Minkowski problem

In this talk, I will talk about the polar Orlicz-Minkowski problem, which is closely related to but quite different from the problem of finding the Orlicz-Petty bodies. In particular, I will explain how the polar Orlicz-Minkowski problem was developed. Moreover, the existence, uniqueness, and continuity of solutions to this problem will be discussed.

This talk is based on a joint paper with Luo and Zhu.