Contextuality is a special feature of quantum systems. Originally it is expressed in the form of no-go theorems of Kochen-Specker, and violation of Bell inequalities. This fundamental property of quantum systems, which turns out to be responsible for speed-up in quantum computers, has been under intense investigation by the quantum computing community. In this work, joint with Daniel Sheinbaum, I will describe a topological approach to contextuality that uses classifying spaces, fundamental objects in algebraic topology. Physically relevant quantities are interpreted as classes in the cohomology and the twisted $K$-theory of the space.