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Uniquely Ergodic C^*-Dynamical Systems for the noncommutative 2-torus

Consider a uniquely ergodic C^*-dynamical system $(\mathfrak{A}, \Phi)$ based on a identity-preserving $*$-endomorphism Φ of the unital C*-algebra $\mathfrak{A}$. We can prove the uniform convergence of Cesaro averages

$$M_{a,\lambda}(n) := \frac{1}{n}\sum_{k=0}^{n-1}\lambda^{-k}\Phi^k(a), \quad a \in \mathfrak{A},$$

for all values λ in the unit circle $\mathbb{T}$, which are not eigenvalues corresponding to "measurable non-continuous" eigenfunctions. This result generalizes an analogous one, known in commutative ergodic theory, which turns out to be a combination of the Wiener-Wintner theorem and the uniformly convergent ergodic theorem of Krylov and Bogolioubov. We also present counterexamples based on the tensor product construction, for which the above average does not converge even in the $*$-weak topology, for some $a \in \mathfrak{A}$ and $\lambda \in \mathbb{T}$.

It would however be desirable to produce more general examples than those (perhaps non trivial) based on the tensor product construction, for which the average $M_{a,\lambda}$ corresponding to some peripheral eigenvalue $\lambda \in \mathbb{T}$ fails to converge for some element $a \in \mathfrak{A}$. It is done as in the classical case, by defining the noncommutative extension of the Anzai skew product on the noncommutative 2-torus $\mathbb{A}_\alpha$ $(2\pi\alpha$ being the deformation angle), and show that, still in these cases, there exist elements $a \in \mathbb{A}_\alpha$ and $\lambda \in \mathbb{T}$ for which the average $M_{a,\lambda}$ does not converge.