Equivariant Methods in Differential and Algebraic Geometry
Méthodes équivariantes en géométrie différentielle et algébrique
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ALEJANDRO ADEM, University of British Columbia

Twisted equivariant K-theory of compact Lie group actions with maximal rank isotropy

We consider twisted equivariant K–theory for actions of a compact Lie group G on a space X where all the isotropy subgroups are connected and of maximal rank. We show that the associated rational spectral sequence à la Segal has a simple E_2-term expressible as invariants under the Weyl group of G. Namely, if T is a maximal torus of G, they are invariants of the $\pi_1(X^T)$-equivariant Bredon cohomology of the universal cover of X^T with suitable coefficients. In the case of the inertia stack ΛY this term can be expressed using the cohomology of Y^T and algebraic invariants associated to the Lie group and the twisting. A number of calculations will be provided; in particular, we recover the rational Verlinde algebra when Y is a point. This is joint work with José Manuel Gómez and José María Cantarero.

TOM BAIRD, Memorial University of Newfoundland

E-polynomials of character varieties associated to a real curve

Given a Riemann surface Σ denote by $M_n(\mathbb{F}) := Hom_\xi(\pi_1(\Sigma), GL_n(\mathbb{F}))/GL_n(\mathbb{F})$ the ξ-twisted character variety for $\xi \in \mathbb{F}$ an n-th root of unity. An anti-holomorphic involution τ on Σ induces an involution on $M_n(\mathbb{F})$ such that the fixed point variety $M_n^\tau(\mathbb{F})$ can be identified with the character variety of "real representations" for the orbifold fundamental group $\pi_1(\Sigma, \tau)$. When $\mathbb{F} = \mathbb{C}$, $M_n^\tau(\mathbb{C})$ is a complex Lagrangian submanifold, a.k.a an ABA-brane.

The E-polynomial of $M_n(\mathbb{C})$ was determined by Hausel and Rodriguez-Villegas by counting points in $M_n(\mathbb{F}_q)$ for finite fields $\mathbb{F}_q$. I will show how the same methods can be used to calculate a generating function for the E-polynomial of $M_n^\tau(\mathbb{C})$ using the representation theory of $GL_n(\mathbb{F}_q)$. This is part of an ongoing collaboration with Michael Lennox Wong.

ANDERS BUCH, Rutgers

Positivity of minuscule quantum K-theory

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The quantum K-theory ring of a flag variety is a generalization of the quantum cohomology ring that encodes information about the arithmetic genera of Gromov-Witten varieties in its structure constants. I will speak about a proof that these structure constants have alternating signs for minuscule flag varieties, including Grassmannians of type A, maximal orthogonal Grassmannians, even dimensional quadrics, and two exceptional flag varieties. This is joint work with Chaput, Mihalcea, and Perrin.

JEFFREY CARLSON, University of Toronto

The equivariant K-theory of a cohomogeneity-one action

We compute the equivariant K-theory ring of a cohomogeneity-one action of a compact, connected Lie group on a smooth manifold. This being by definition the case when the orbit space is one-dimensional, it can be seen as a natural next case after that of a transitive action.

Concrete expressions analogous to the cohomological case (due to the speaker, Goertsches, He, and Mare) only arise in the case when the fundamental group of a stabilizer is free abelian. The computation of the additive structure is mainly representation theory and Lie theory, and the multiplicative structure, surprisingly, follows from the Mayer–Vietoris sequence.

PETER CROOKS, Northeastern University
Kostant-Toda lattices and invariant theory

Toda lattices play a distinguished role in both the classical and modern theories of completely integrable systems, and they are fruitfully studied at the interface of symplectic geometry and representation theory. One crucial aspect of this study is Kostant's Lie-theoretic realization of the open Toda lattice, which one sometimes calls the Kostant-Toda lattice. This construction invokes Kostant's prior works on invariant theory, especially his results on regular Slodowy slices and the structure of the adjoint quotient.

I will discuss invariant-theoretic aspects of the Kostant-Toda lattice, emphasizing two recent developments. The first is a partial compactification of the Kostant-Toda lattice by means of Hessenberg varieties, Slodowy slices, and Mishchenko-Fomenko algebras, and it represents joint work with Hiraku Abe. The second development concerns a Toda-type integrable system on the *universal centralizer*, a hyperkähler manifold arising in certain representation-theoretic contexts.

CHI-KWONG FOK, The University of Auckland
Twisted K-theory and extended Verlinde algebra

In a series of recent papers, Freed, Hopkins and Teleman put forth a deep result which identifies the twisted K-theory of a compact Lie group G with the representation theory of its loop group LG. Under suitable conditions, both objects can be enhanced to the Verlinde algebra, which appears in mathematical physics as the Frobenius algebra of a certain topological quantum field theory, and in algebraic geometry as the algebra encoding information of moduli spaces of G-bundles over Riemann surfaces. The Verlinde algebra for G with nice connectedness properties has been well-known. However, explicit descriptions of such for disconnected G are lacking. In this talk, I will discuss various aspects of the Freed-Hopkins-Teleman Theorem and partial results on an extension of the Verlinde algebra of a simply-connected compact Lie group arising from a disconnected Lie group. The talk is based on work in progress joint with David Baraglia and Varghese Mathai.

MATTHIAS FRANZ, University of Western Ontario
The number of connected orbit types in a G-manifold

The orbits of a compact Lie group G acting on a manifold X are classified by conjugacy classes of closed subgroups of G. The slice theorem implies that there are only finitely many orbit types if X is compact. Mann showed in 1962 that the same conclusion holds if X is orientable and of finite type.

I will present an analogous theorem for connected orbit types, where one only looks at the isotropy Lie algebras: The number of connected orbit types is finite if X has finite Betti numbers. The proof rests on fundamental properties of a suitably defined equivariant homology theory.

REBECCA GOLDIN, George Mason University
Schubert Calculus, Schubert Operators, and Positivity

Schubert calculus is the study has "positivity properties" in several rings associated to the homogeneous spaces, notably equivariant cohomology and equivariant K-theory. We show how one can get new and old formulas for the structure constants in these rings. We introduce new operators whose coefficients compute Schubert structure constants (in a manifestly polynomial, but not positive, way), resulting in a formula that generalizes the positive AJS/Billey formula. Our proof involves Bott-Samelson manifolds and in particular, the cohomology basis dual to the homology basis of classes of sub-Bott-Samelson manifolds.

MARK HAMILTON, Mount Allison University
Integral integral affine geometry, quantization, and Riemann-Roch.

Let B be a compact integral affine manifold. If the coordinate changes are not only affine but also preserve the lattice $\mathbb{Z}^n$, then there is a well-defined notion of "integral points" in B, and we call B an *integral integral affine manifold*. I will discuss the relation of integral integral affine structures to quantization and some associated results, in particular the fact that for a regular Lagrangian fibration $M \to B$, the Riemann-Roch number of M is equal to the number of "integral points" in B. Along the way we encounter the fact that the volume of B is equal to the number of integral points, a simple claim from "integral integral affine geometry" whose proof turns out to be surprisingly tricky. This is joint work with Yael Karshon and Takahiko Yoshida.

DEREK KREPSKI, University of Manitoba

An analogue of Kostant's formula for Lie group-valued moment maps

Let $P \to M$ be a prequantum circle bundle with connection over a symplectic manifold M. Let G be a compact Lie group. Given a Hamiltonian G-action on M, Kostant gave a formula for lifting the infinitesimal $\mathfrak{g}$-action on M to one on P that preserves the connection. This talk presents an analogue of Kostant's formula in the setting of quasi-Hamiltonian G-spaces with G-valued moment map. This is joint work with Jennifer Vaughan.

JEREMY LANE, University of Geneva

Volume exhausting, T-equivariant symplectic embeddings of toric manifolds into regular coadjoint orbits

Let K be a compact connected Lie group and let $\mathcal{O}_\lambda$ denote the coadjoint orbit of K parameterized by an element λ in the positive Weyl chamber. In several cases, it is known that there exist dense symplectic embeddings of symplectic toric manifolds into $\mathcal{O}_\lambda$. If $K = U(n)$, then, for all λ, one obtains such embeddings from action-angle coordinates for the Gelfand-Zeitlin integrable systems. For arbitrary K compact and connected, if λ is a scalar multiple of an integral weight, then such embeddings can be constructed by toric degeneration and gradient-Hamiltonian flow (cf. Harada and Kaveh). It remains to study the case where λ is not a scalar multiple of an integral weight.

In current work with Anton Alekseev, Benjamin Hoffman, and Yanpeng Li, we show that for K semisimple and any regular coadjoint orbit $\mathcal{O}_\lambda$, one can construct a family of volume exhausting symplectic embeddings of toric manifolds into $\mathcal{O}_\lambda$. Moreover, these embeddings are equivariant with respect to a Hamiltonian action of a maximal torus of K. Our construction combines elements of Poisson-Lie theory, cluster algebras, and a scaling limit of Poisson structures called "tropicalization." In this talk I will endeavour to explain these results as well as our hopes for future work.

YIANNIS LOIZIDES, Pennsylvania State University

Quasi-polynomials, asymptotics and [Q,R]=0

We study families (parametrized by a positive integer, k) of distributions associated to a class of piecewise quasi-polynomial functions on a lattice. These distributions admit an asymptotic expansion in k, and we study to what extent the original family can be recovered from its asymptotic expansion. This is joint work with P-E. Paradan and M. Vergne. I will discuss some applications to symplectic geometry, where the piecewise quasi-polynomial functions arise as multiplicity functions for the equivariant index of a Dirac operator twisted by the k^{th} power of a pre-quantum line bundle.

JENNA RAJCHGOT, University of Saskatchewan

Grobner bases for certain type C Kazhdan-Lusztig ideals

A Kazhdan-Lusztig variety is an intersection of a Schubert variety with an opposite Schubert cell in a flag variety. By a theorem of Kazhdan and Lusztig, it is essentially equivalent to study certain local questions on Schubert varieties by studying the corresponding questions on Kazhdan-Lusztig varieties.

Alexander Woo and Alexander Yong carried out a computational-algebraic study of Kazhdan-Lusztig varieties in type A. They produced Grobner bases for their defining ideals, obtained singularity results, and produced combinatorial formulas for their K-polynomials (equivariant K-classes).

After recalling some background, I will discuss work in progress with Laura Escobar, Alex Fink, and Alexander Woo on computational-algebraic properties of certain type C Kazhdan-Lusztig varieties.

STEVEN RAYAN, University of Saskatchewan
Linearizations of character varieties of curves: geometry and mirror symmetry

To a tamely-punctured algebraic curve with generic positive real weights, one can attach a character variety that can be identified with a multiplicative Nakajima variety of a certain shape. The linearization of this character variety is an ordinary Nakajima quiver variety, called a "hyperpolygon space" in certain instances. The weights, which are stability parameters (equivalently, levels sets of moment maps), produce a singular hyperkaehler variety when driven to zero. This variety at infinity is stratified in a particular way by "edge collapse". Natural questions include the following: Can the singular hyperkaehler metric be written explicitly? Does this variety arise from a finite subgroup of a Lie group (in the sense of McKay)? Is the mirror of this variety well understood as, say, a Coulomb branch and what are its "branes" (in the sense of Kapustin-Witten)? I will comment on all of these, as part of joint work with each of Laura Schaposnik and Hartmut Weiss.

EVAN SUNDBO, University of Saskatchewan
The Geometry of Twisted Cyclic Quiver Varieties

Cyclic Higgs bundles find uses in various aspects of non-abelian Hodge theory (where they are manageable enough to do examples with) and have proven to be potent problem-solving tools in other areas as well. Starting from the world of twisted quiver representations, we investigate the geometry of moduli spaces of L-twisted cyclic Higgs bundles, being particularly concrete when the underlying Riemann surface is the projective line $\mathbb{P}^1$.

This is joint work with Steven Rayan.

JORDAN WATTS, Central Michigan University
Classifying Spaces for Diffeological Groups

Fix an irrational number A, and consider the action of the group of pairs of integers on the real line defined as follows: the pair (m, n) sends a point x to $x + m + nA$. Since the orbits of this action are dense, the quotient topology on the orbit space is trivial, and continuous real-valued functions are constant. Can we give the space any type of useful "smooth" group structure?

The answer is "yes": its natural diffeological group structure. It turns out this group is not just some pathological example, but has many interesting associated structures, and is of interest to many areas of mathematics. In particular, it shows up in geometric quantisation and the integration of certain Lie algebroids as the structure group of certain principal bundles, the main topic of this talk.

We will perform Milnor's construction in the realm of diffeology to obtain a diffeological classifying space for a diffeological group G, such as the irrational torus. After mentioning a few hoped-for properties, we then construct a connection 1-form on the G-bundle $EG \to BG$, which will naturally pull back to a connection 1-form on sufficiently nice principal G-bundles. We then look at what this can tell us about irrational torus bundles.

KIRILL ZANOULLINE, University of Ottawa
Localized Landweber-Novikov operations on generalized cohomology

Cohomological operations on generalized algebraic cohomology theories (e.g. Steenrod, Adams, Landweber-Novikov) have been extensively studied during the past decade (Brosnan, Levine, Merkurjev, Vishik). They turned out to be extremely useful in generating interesting rational cycles in higher codimension (e.g. idempotents or 0-cycles on twisted flag varieties), hence, in computing various geometric invariants of torsors (incompressibility, canonical dimension, torsion, motivic decomposition type, etc.).

In the present talk, we explain how to extend the Landweber-Novikov operations on algebraic cobordism to the setup of equivariant generalized cohomology theories via the localization techniques of Kostant-Kumar. The operations we obtain we call localized operations. These operations can be viewed as operations on global sections of the so called structure sheaves on moment graphs (corresponding to arbitrary Coxeter groups). They satisfy several natural properties, e.g. they commute with characteristic map and restrict to usual Landweber-Novikov operations.

CHANGLONG ZHONG, SUNY-Albany

On the K-theoretic stable bases

In this talk I will recall Maulik-Okounkov's definition of K-theoretic stable bases. I will then talk about their restriction formulas. Note that stable bases depend on the alcoves of the group of characters of the torus. Stable bases in terms of different alcoves are related by the so-called wall crossing R-matrices, studied by Okounkov. I will also talk about such matrices. This is joint work with Changjian Su and Gufang Zhao.