Categorical Approaches to Topology and Geometry
Approches catégoriques de la topologie et de la géométrie

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KRISTINE BAUER, University of Calgary

The Kleisli category of a pseudomonad for chain complexes

The construction of the Kleisli category of a 2-monad acting on a 2-category is well known and mostly harmless. For each structure or axiom needed to produce such a Kleisli category, it is possible to consider a weak version. For example, one could consider bicategories rather than 2-categories, a pseudo- or lax version of the monad, or simply a weakening of axioms like associativity. Many authors have studied Kleisli constructions in the presence of one sort of weakening or another (Cheng, Gordon, Hyland, Lack, Marmolejo, Niefeld, Powers, Street are a few examples amongst many). In this expository talk, I will consider various weak constructions with an eye towards a particular example arising from the category of all abelian categories, and the chain complex monad. This example is related to joint work with Brenda Johnson and Sarah Yeakel.

MARZIEH BAYEH, Dalhousie University

Orbit Category and The Category of Orbit Classes

For a group G, the orbit category of G, denoted by $\mathcal{O}_G$, is the category of orbit types and equivariant maps. In this talk I will introduce the category of orbit classes for a G-space X, and will study the relation between this category and the orbit category.

ROBIN COCKETT, University of Calgary

Hyperconnections

Functors between join restriction categories admit a factorization into localic functors (bijective on objects and meet preserving) followed by hyperconnected functors (bijective on the locales of restriction idempotents) - for the basic localic/hyperconnected factorization on mere restriction categories.

A partite category (a *partite* internal category has its objects and arrow partitioned into many objects) internal to a join restriction category $\mathbb{B}$ induces, by considering partial sections of the domain maps, an external join restriction category which sits over $\mathbb{B}$ by a hyperconnected functor. Conversely an external join restriction category over $\mathbb{B}$ (where the latter must be assumed to have all gluings) induces a source étale partite category internal to $\mathbb{B}$. This correspondence may be completed to a Galois adjunction between join restriction categories over $\mathbb{B}$ and partite categories internal to $\mathbb{B}$ with cofunctors. The adjunction specializes to an equivalence between hyperconnections over $\mathbb{B}$ and source étale partite categories internal to $\mathbb{B}$.

This phenomenon occurs in many different places in mathematics (often specialized to groupoids). In particular, as all join restriction categories have a hyperconnected fundamental functor to the category of locales (with partial maps) one can conclude that join restriction categories (with join functors) correspond precisely to source étale partite categories internal to locales (with cofunctors). From algebraic geometry, considering schemes as a join restriction category with gluings, the identity functor on schemes induces an internal partite category: the object of morphisms from the affine scheme R to $\mathbb{Z}[x]$ is then exactly the structure sheaf of R.

HELLEN COLMAN, Wright College

Equivariant motion planning

Consider the space X of all possible configurations of a mechanical system. A motion planning algorithm assigns to each pair of initial and final states $(a, b) \in X \times X$, a continuous motion of the system starting at a and ending at b.

Topological complexity is an integer $TC(X)$ reflecting the complexity of motion planning algorithms for all systems having X as their configuration space. Roughly, $TC(X)$ is the minimal number of continuous rules which are needed in a motion planning algorithm. This invariant was introduced by Farber in 2002 and is closely related to the classical Lusternik-Schnirelmann category.

In recent years, several versions of topological complexity aimed at exploiting the presence of a group G acting on the configuration space have appeared. We will present several approaches to describing equivariant topological complexity variants in the context of the bicategory of orbifold translation groupoids and will show that $TC^G(X)$ is invariant under Morita equivalence.

GEOFF CRUTTWELL, Mount Allison University
Curvature and torsion without negatives

A tangent category abstracts settings in which each object has an associated "tangent bundle". Examples range from standard settings for differential geometry (smooth manifolds, infinite-dimensional manifolds, synthetic differential geometry) to algebraic geometry (schemes) to computer science (Cartesian differential categories) and to homotopy theory (Abelian functor calculus, potentially Goodwillie functor calculus). One advantage of this abstraction is that it captures settings which have a tangent bundle but may not have negation of tangent vectors.

In the past several years, there has been a lot of development of differential geometric ideas within a tangent category: definitions have been given for the Lie bracket, vector bundles, connections, differential forms, and de Rham cohomology. However, in a few cases, these definitions have required the assumption that one could negate tangent vectors, reducing the applicability of those definitions from the full range of settings mentioned above.

In particular, the previously given definitions of curvature and torsion in this setting required negatives. In this talk, I'll show how to define curvature and torsion in this abstract setting without requiring the existence of negatives, leading to their applicability in a wider variety of examples.

DARIEN DEWOLF, St. Francis Xavier University
The equivalence of ordered groupoids and left cancellative categories using double categories

Lawson gave a correspondence between left cancellative categories and ordered groupoids (groupoids equipped with a partial order on its arrows and that have a notion of restricting/corestricting to a smaller domain/codomain). Lawson and Steinberg then introduce Ehresmann topologies and give a correspondence between Ehresmann topologies on ordered groupoids and Grothendieck topologies on left cancellative categories, and prove that any etendue is equivalent to the category of sheaves on some Ehresmann site (an ordered groupoid equipped with an Ehresmann topology).

By considering ordered groupoids as double categories, we are able to extend Lawson's correspondence to an adjoint equivalence of 2-categories between the 2-category of left cancellative categories and the 2-category of ordered groupoids. Making use of this equivalence, we can then state a comparison lemma for Ehresmann sites, which classifies functors between Ehresmann sites that induce an equivalence between their categories of sheaves.

This is joint work with Dorette Pronk.

JONATHAN GALLAGHER, Dalhousie University
Etale Subobject classifiers in SDG and tangent categories

One aspect of the success of topos theory in geometry is the ability to use a rich internal language to describe geometric constructions. The power of the internal logic of a topos stems from existence of a subobject classifier. However, in settings for abstract differential geometry, tangent categories, one often does not have a subobject classifier.

This talk will discuss work to understand a weaker condition for tangent categories called an étale subobject classifier. Indeed, having an étale subobject classifier in a tangent category is weaker than being a quasitopos. We will explore the logic required to obtain an étale partial map classifier from an étale subobject classifier. We will also explore the resulting logic in the étale subobject fibration which has connectives $\wedge, \top, \exists, \vee, \bot$.

We will then discuss more generally the coherence of partial cartesian closed tangent categories, making use of recent work on tangent categories to characterize tangent bundles abstractly. We will discuss what it means for partial maps to be classified in this setting. To provide a less SDG example, we will discuss partial maps between convenient vector spaces.

CHRIS KAPULKIN, University of Western Ontario
Cubical models of higher category theory

Cubical sets provide an alternative to simplicial sets as a combinatorial model for spaces a.k.a. ∞-groupoids. Joyal showed that simplicial sets also carry a model structure presenting $(\infty, 1)$-categories. However, the cubical counterpart of his result has not been established.

The goal of this project is to develop workable foundations of $(\infty, 1)$-category theory in the category of cubical sets. To this end, in joint work with Christian Sattler and Liang Ze Wong, we define four model structures on categories of (marked and unmarked) cubical sets and explore some of their properties.

This part 2 of a 2-part series given by Chris Kapulkin and Zach Lindsey.

MICHAEL LAMBERT, Dalhousie University
A Site for Continuous 2-Group Actions

The category of continuous group actions for a fixed topological group is well-known to be Grothendieck topos. In this talk, we shall outline our work on constructing an appropriate higher site for continuous actions of topological 2-groups.

JS LEMAY, University of Oxford
The Poincaré Lemma for Codifferential Categories with Antiderivatives

The Poincaré Lemma, named after the french mathematician Henri Poincaré, states that for a contractible manifold: a closed differential form is exact. In particular this implies that the de Rham complex of $\mathbb{R}^n$ is contractible (or equivalently split exact). Similarly, the algebraic version of the Poincaré Lemma states that the algebraic de Rham complex (the one built from Kähler differentials) of a polynomial ring $\mathbb{R}[x_1, ..., x_n]$ is contractible. In this talk we provide a Poincaré Lemma for codifferential categories with antiderivatives by showing that the de Rham complex of a free S-algebra is contractible, where S is the monad of the codifferential category. Taking S to be the free symmetric algebra monad results in the algebraic Poincaré Lemma, while taking S to be the free $\mathcal{C}^\infty$-ring monad gives the classical Poincaré Lemma.

ZACH LINDSEY, University of Western Ontario
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RORY LUCYSHYN-WRIGHT, Brandon University
Functional distribution monads and τ-additive measures

The article [1] defines a categorical framework for algebraic dualization processes that give rise to various spaces of measures, distributions, filters, closed subsets, compacta, and so forth. In this framework, the latter are all captured as instances of a general construction that begins with a $\mathcal{V}$-enriched algebraic category $\mathcal{A}$, with a suitable object to play the role of 'dualizer', and produces an associated monad on $\mathcal{V}$ called the *functional distribution monad*.

In this talk, we will show that by taking as $\mathcal{V}$ the category of convergence spaces and as $\mathcal{A}$ the category of convex spaces internal to $\mathcal{V}$, with the unit interval as dualizer, the induced functional distribution monad gives rise to the notion of *τ-additive* (or *τ-smooth*) *probability measure* on Tikhonov spaces. Thus the resulting monad captures a wide class of settings in topological measure theory, including not only Radon probability measures on locally compact spaces but also Borel probability measures on Polish spaces. In proving our result, we establish a connection between τ-additive measures and *continuous convergence*, and we prove integral representation theorems for τ-additive measures that are formulated in terms of the cartesian closed structure of the category of convergence spaces.

[1] R. B. B. Lucyshyn-Wright, Functional distribution monads in functional-analytic contexts. *Advances in Mathematics* 322 (2017), 806–860.

BEN MCADAM, University of Calgary
Involution Algebroids

We define involution algebroids which generalize Lie algebroids to the abstract setting of tangent categories. As a part of this generalisation, the Jacobi identity which appears in classical Lie theory is replaced by an identity similar to the Yang-Baxter equation. Every classical Lie algebroid has the structure of an involution algebroid and every involution algebroid in a tangent category admits a Lie bracket on the sections of its underlying bundle.

This is joint work with Matthew Burke.

CURRAN MCCONNELL, Dalhousie University
Combinatorics of spaces of trees: an application of topology to phylogenetics

Various metrics are used in phylogenetics to study sets of evolutionary trees generated from gene sequences. We want to use some of these metrics to consider what persistent homology might be able to contribute to the study of these trees. Our "data points" are points in the space of all trees with n leaves, where n is the number of species considered. We will consider the family of edge complexes, indexed by a sequence of real numbers ϵ_i, obtained by adding an edge between two data points if their distance is less than or equal to ϵ_i. This gives us a filtration of the $((2n-3)!!-1)$-simplex with interesting homological properties, in particular for the quartet distance. Any given data set will give rise to a subsimplex of this $((2n-3)!!-1)$-simplex and a subfiltration. Understanding the properties of the surrounding simplicial complex and its filtration will be important in understanding which features are truly features of the data set we are considering. In this talk I will discuss the features of these simplicial complexes for low values of n and I will present some conjectures for what this means for higher values of n.

DORETTE PRONK, Dalhousie University
Nice properties of the bicategory of orbigroupoids

Based on the presentation of orbigroupoids as a bicategory with small Hom-categories and canonical representatives for its 2-cells as discussed in Laura Scull's talk, I will present a continuous version of the assumptions required. For instance, where listings of certain arrows are required, I show that these listings can be chosen in a continuous way. I will use this to prove properties of the mapping groupoids for orbigroupoids. This is joint work with Laura Scull from Fort Lewis College.

JONATHAN SCOTT, Cleveland State University
Wasserstein distance for generalized persistence modules and abelian categories

In persistence theory and practice, measuring distances between modules is central. The Wasserstein distances are the standard family of L^p distances (with $1 \leq p \leq \infty$) for persistence modules. We give an algebraic formulation of these distances. For $p = 1$ the distance generalizes to abelian categories and for arbitrary p it generalizes to Krull-Schmidt categories. These distances may be useful for the computation of distance between generalized persistence modules. This is joint work with Peter Bubenik and Donald Stanley.

LAURA SCULL, Fort Lewis College
Defining Bicategories of Fractions with Small Hom Sets

This work grew out of the question of building a mapping object for orbifolds. The bicategory of orbifolds is a bicategory of fractions of proper etale groupoids with respect to the class of essential equivalences. A priori, the hom categories in this category are extremely large and somewhat mysterious, since the essential equivalences over a given groupoid form a proper class.

I will discuss categorical conditions which allow us to better understand constructions like these. We develop weaker conditions for a bicalulus of fractions to exist, and show how this can be used to pass to a small subclass of arrows to be inverted. Time permitting, I will talk about how to use pseudo pullbacks to simplify the 2-cell constructions and compositions in the resulting bicategory of fractions.

This is joint work with D. Pronk at Dalhousie University.

JOEL VILLATORO, KU Leuven
Geometric structures on differentiable stacks

Differentiable stacks are a formalism for studying geometric objects which arise as singular quotients of smooth manifolds. These categorical objects can be studied concretely via their associated Lie groupoids. In this talk I will explain a general approach to studying differentiable stacks which are associated to Lie groupoids that come with additional structure (such as symplectic or analytic structures).

JORDAN WATTS, Central Michigan University
Bredon Cohomology for Transitive Groupoids

Given a topological group G acting on a space X, Bredon cohomology is a useful tool for picking out the cohomology of the quotient X/G, as well as fixed sets X^H, where H is a subgroup of G. In this talk, we extend Bredon cohomology to actions of transitive groupoids. [Joint work with Carla Farsi and Laura Scull.]

SETH WOLBERT, University of Manitoba
Fibrations as presentations of actions on stacks

Given a Lie group G and a stack $\mathcal{X}$ over the site of smooth manifolds, *an action of G on $\mathcal{X}$* is a map of stacks $a : G \times \mathcal{X} \to \mathcal{X}$ for which the standard action axiom diagrams are required only to commute up to 2-isomorphism. One may define the action of a Lie groupoid G on $\mathcal{X}$ similarly as a weakened version of a standard groupoid action.

In this talk, I will explain how if $\mathcal{X}$ is a differentiable stack presented by some Lie groupoid H, the data of an action of G on $\mathcal{X}$ can be repackaged as a Lie groupoid fibration $\pi : A \to G$ with kernel groupoid H. As fibrations are relatively common in the study of Lie groupoids, I will (time permitting) be able to give plenty of examples of the transition between Lie groupoid fibrations and stack actions, including examples related to gerbes, VB-groupoids, and flows of vector fields on stacks.