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On the average value of a function of the residual index

For a prime p and a positive integer a relatively prime to p, we denote $i_a(p)$ as the index of the subgroup generated by a in the multiplicative group $\mathbb{F}_p^\times$. Under certain conditions on the arithmetic function $f(n)$, we prove that the average value of $f(i_a(p))$, as a and p vary, is

$$\sum_{d=1}^{\infty} \frac{g(d)}{d\varphi(d)},$$

where $g(n) = \sum_{d|n} \mu(d) f(n/d)$ is the Möbius inverse of f and $\varphi(n)$ is the Euler function. In the special case of $f(n) = \log n$, our result establishes, unconditionally, on average over a, a conjecture proposed by Bach, Lukes, Shallit, and Williams, and also stated by Fomenko. This is joint work with Adam Felix.